Indirect Methods for Nuclear Astrophysics

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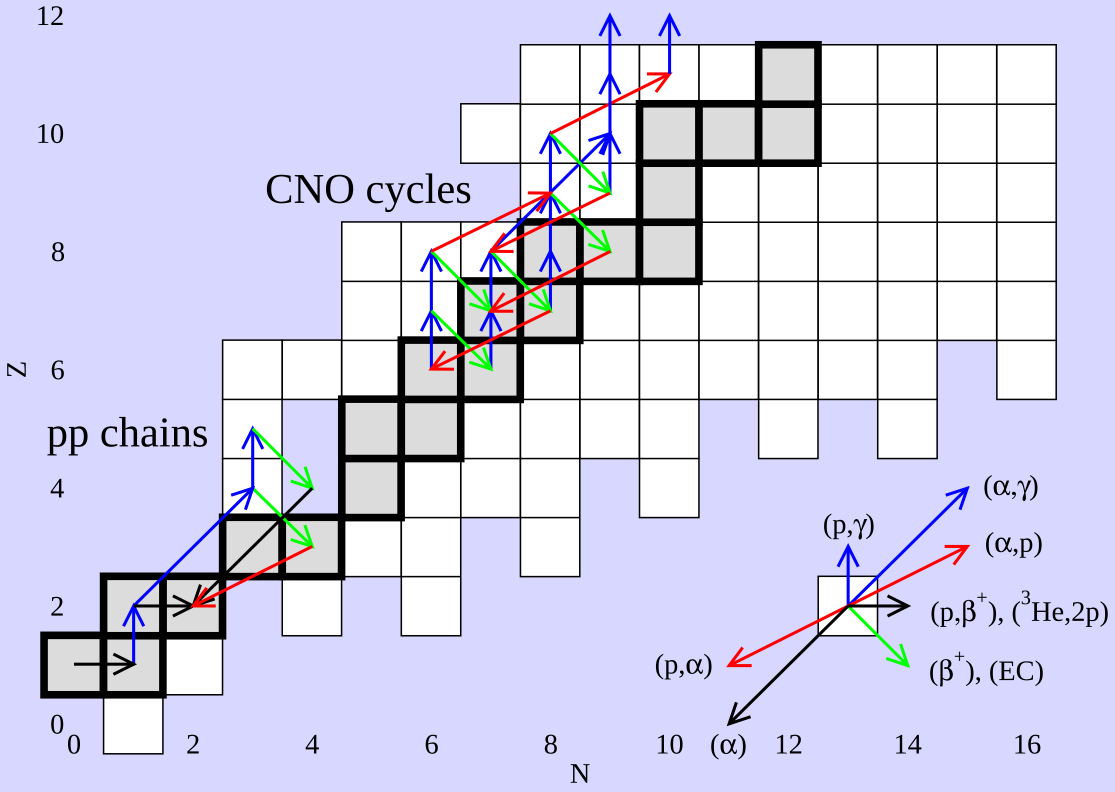
3$^{\mathrm{rd}}$ Russbach Workshop on Nuclear Astrophysics

(VISTARS - JINA)

Outline

- **Motivation**
  - nuclear reactions of astrophysical interest & direct experiments
  - reactions rates, S factor & electron screening
- **Indirect Methods**
  - overview & general characteristics
  - reaction theory & DWBA
  - Coulomb Dissociation method
  - transition matrix elements & asymptotics of wave functions
  - ANC method
  - effects of interaction in continuum states
  - Trojan-Horse method
- **Conclusions**

Nuclear Reactions of Astrophysical Interest



nuclear astrophysics

nuclear reaction rates are basic input in many astrophysical models (primordial nucleosynthesis, stellar evolution, novae, supernovae, . . .) for various processes (pp chains, CNO cycles, s, r, p, rp, . . .)

- radiative capture/dissociation reactions with charged particles: (p,γ), (α,γ), . . .
- direct nuclear reactions: (p,α), (α,p), . . .
- weak interaction reactions: β^+, β^-, EC

ideally:

direct measurement

of cross sections

at relevant energies

BUT. . .

Reaction Rate

astrophysical environment: nuclei in hot plasma

⇒ temperature-dependent distribution of velocities

- Maxwellian-averaged reaction rate $(b + c \to \dots)$

$$r_{bc} = \frac{\varrho_b \varrho_c}{1+\delta_{bc}} \langle \sigma v \rangle$$

with densities ϱ_b, ϱ_c and

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu}} \int_0^\infty \sigma(E)\ E\ e^{-\frac{E}{k_B T}} \frac{dE}{(k_B T)^{3/2}}$$

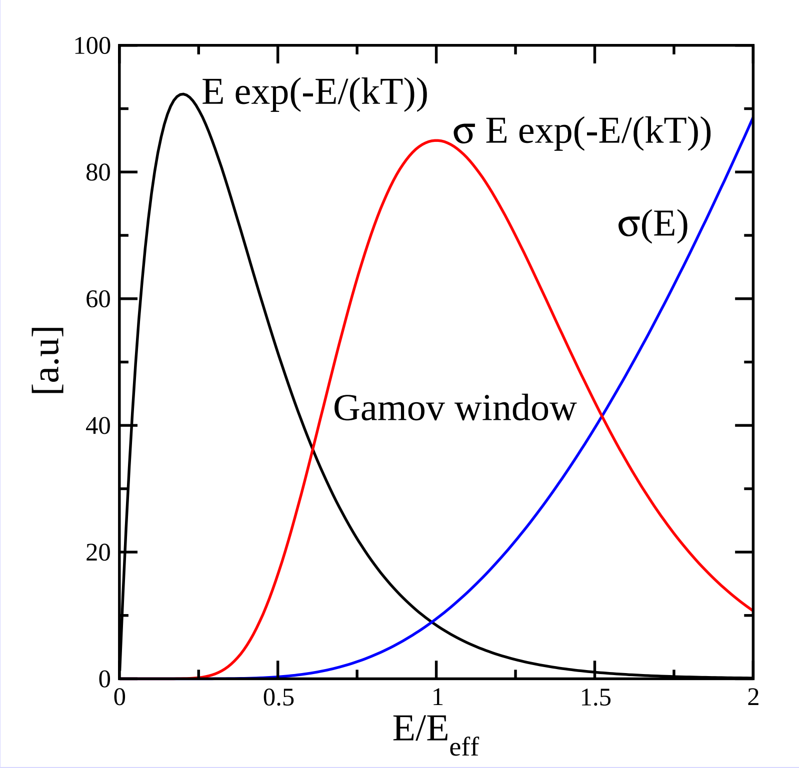
⇒ cross sections needed in Gamov window with effective energy

$$E_{\text{eff}} = 0.1220\, \mu^{1/3} (Z_b Z_c T_9)^{2/3} \text{ MeV}$$

and width

$$\Delta E = 0.2368\, \mu^{1/6} (Z_b Z_c)^{1/3} T_9^{5/6} \text{ MeV}$$

with temperature T_9 in 10^9 K
and reduced mass μ in amu



reaction	E_{eff} [keV]	$\sigma(E_{\text{eff}})$ [pb]
^{3}He(^{3}He,2p)^{4}He	22.0	1.5
^{7}Be(p,γ)^{8}B	18.4	1.5×10^{-3}
^{3}He(α,γ)^{7}Be	23.0	3.0×10^{-5}
^{14}N(p,γ)^{15}O	27.2	2.2×10^{-7}

for $T = 15.5 \times 10^6$ K (center of the sun)

Charged-Particle Reactions

- **Coulomb barrier** in reaction $b + c \rightarrow \ldots$ with charged nuclei b, c
  - ⇒ extremely small cross sections $\sigma(E)$ with strong energy dependence
  - ⇒ astrophysical relevant energies (Gamov peak) usually not accessible
  - ⇒ measurement at higher energies and extrapolation to low energies E

    with help of astrophysical S factor $\boxed{S(E) = \sigma(E) E \exp(2\pi\eta)}$

    Sommerfeld parameter $\eta = Z_b Z_c e^2/(\hbar v_{bc})$
  - ⇒ danger of extrapolation error, missed resonances, bound state tails

- direct measurement very demanding, often unstable nuclei involved

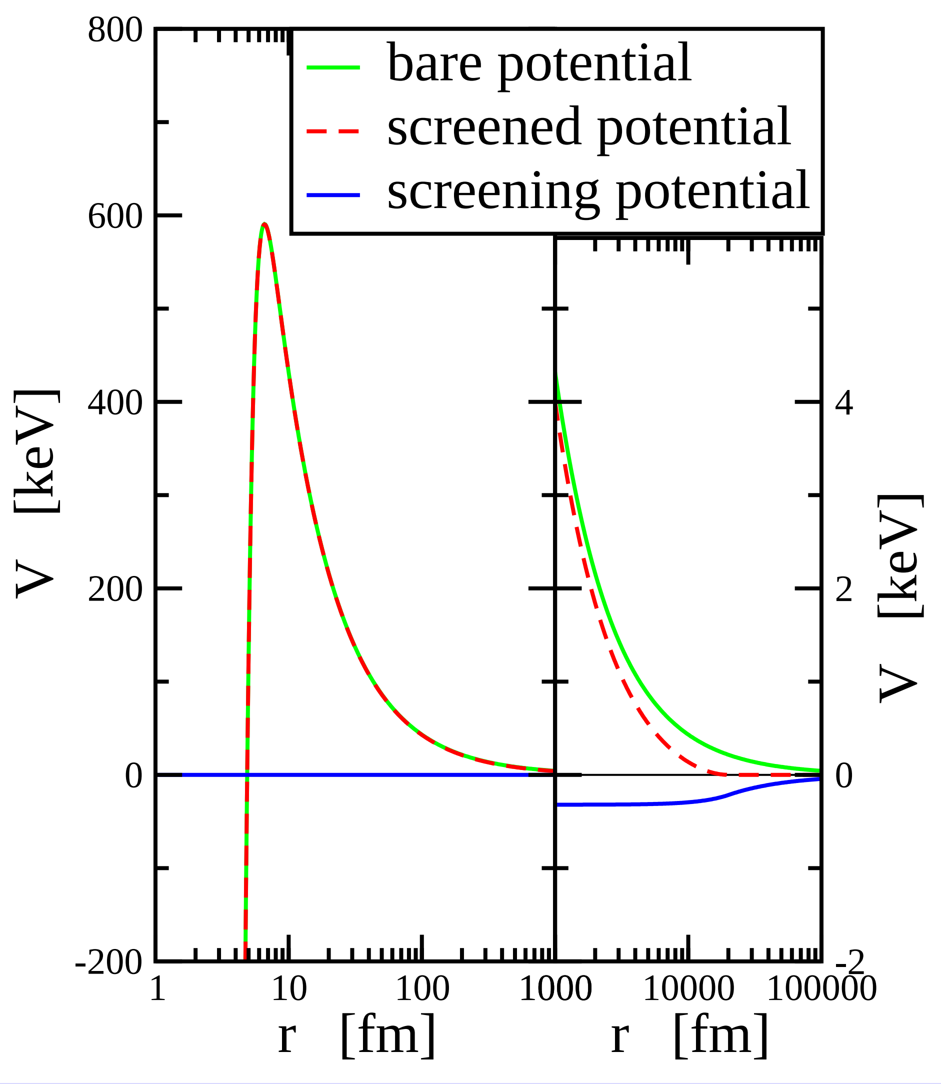
- cross sections of light particle reactions are dominated by non-resonant and only few resonant contributions at small energies

alternative: indirect methods

Electron Screening

electron screening in direct experiments

- reduction of Coulomb barrier by electron cloud of target nucleus
- enhanced cross section at low energies
  $$\sigma_{\rm exp}(E) = \sigma_{\rm bare}(E) f(E)$$
  with $f(E) = \exp(\pi\eta U_e/E)$ and
  electron screening potential energy U_e
- discrepancy between experimental observation and theoretical models, explanation?
- independent information needed from experiments: indirect methods
- stellar conditions: electron screening in plasma



Indirect Methods - CD/ANC/THM

Coulomb dissociation

G. Baur et al.,
NPA 458 (1986) 188

- study inverse of radiative capture reaction $b(x,\gamma)a \;\Leftrightarrow\; a(\gamma,x)b$
- use Coulomb field of target nucleus A as source of photons $a(\gamma,x)b \;\Leftrightarrow\; A(a,bx)A$

$\Downarrow$

absolute S factors
as a function of energy

ANC method

H. M. Xu et al.,
PRL 73 (1994) 2027

- extract asymptotic normalization coefficient of ground state wave function of nucleus a from transfer reactions
- calculate matrix elements for radiative capture reaction $b(x,\gamma)a$

$\Downarrow$

S factor at zero energy

Trojan-Horse method

G. Baur,
PLB 178 (1986) 35

- study three-body reaction $A+a \longrightarrow C+c+b$ with Trojan horse $a=b+x$ and spectator b
- extract cross section of two-body reaction $A+x \longrightarrow C+c$

$\Downarrow$

energy dependence
of S factor

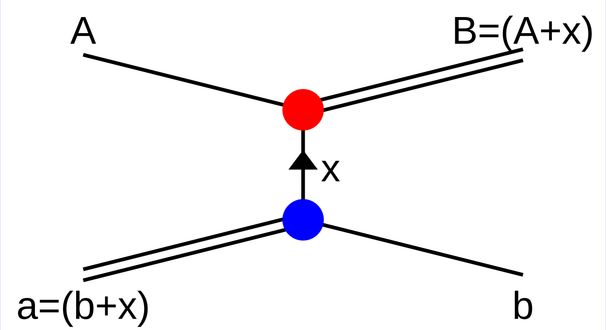
Relation of methods? Problems?

Indirect Methods - General

general characteristics:

- two-body reaction at low-energy is replaced by three-body reaction at "high-energy" with large cross section
  - ○ Coulomb dissociation $\boxed{b(x,\gamma)a \Rightarrow A(a,bx)A}$
  - ○ ANC method $\boxed{b(x,\gamma)a \Rightarrow A(a,B)b}$
  - ○ Trojan-horse method $\boxed{A(x,c)C \Rightarrow A(a,Cc)b}$
- transfer of virtual particle (photon γ or nucleus x)
- relation of cross sections is found with the help of nuclear direct reaction theory
- theoretical approximations essential
  ⇒ factorization of T-matrix elements and cross sections
- study of peripheral reactions
  - – asymptotics of wave functions relevant
  - – selection of kinematical conditions

Cross Section of Transfer Reactions



- **transfer reaction** $A + a \longrightarrow B + b$ with $a = b + x$

  to bound state of $B = A + x$

- **cross section**

$$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}} \left|T_{fi}\right|^2 \delta(E_B + E_b - E_A - E_a - Q) \frac{d^3 p_{Bb}}{(2\pi\hbar)^3}$$

  with T-matrix in prior formulation $T_{fi} = \langle \Psi_{Bb}^{(-)} | V_{Aa}^{(i)} | \exp(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}) \phi_A \phi_a \rangle$

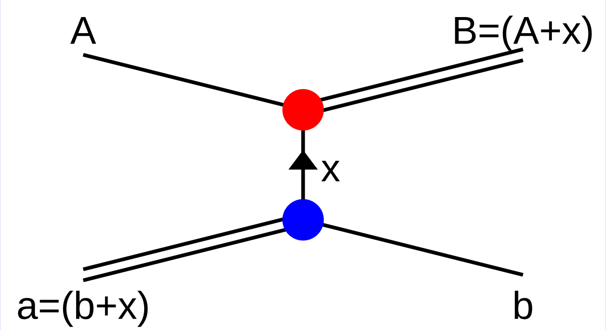
  or in post formulation $T_{fi} = \langle \exp(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}) \phi_B \phi_b | V_{Bb}^{(f)} | \Psi_{Aa}^{(+)} \rangle$

  $\Psi_{Aa}^{(+)}/\Psi_{Bb}^{(-)}$: exact scattering wave functions in initial/final state

  $V_{Aa}^{(i)}/V_{Bb}^{(f)}$: full potentials in initial/final state

- T-matrix element contains essential information on reaction process
- transferred particle x is virtual, i.e. $E_x \neq \frac{p_x^2}{2m_x}$
- idea: two poles in diagram $\Rightarrow$ factorization

Transformation of T-Matrix Elements



- expressions for T-matrix element

$$T_{fi} = \langle \Psi^{(-)}_{Bb} | V^{(i)}_{Aa} | \exp(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}) \phi_A \phi_a \rangle$$ (prior form)

$$T_{fi} = \langle \exp(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}) \phi_B \phi_b | V^{(f)}_{Bb} | \Psi^{(+)}_{Aa} \rangle$$ (post form)

not useful for calculation

- introduce optical potentials U_{ij} $(ij = Aa, Bb)$

and distorted waves $\chi^{(\pm)}_{ij}$ with $(\hat{T}_{ij} + U_{ij})\chi^{(\pm)}_{ij} = E_{ij}\chi^{(\pm)}_{ij}$

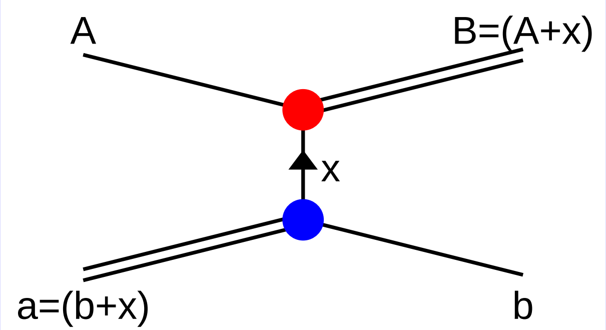
- apply Gell-Mann–Goldberger relation (Phys. Rev. 91 (1953) 398) ⇒

$$T_{fi} = \langle \Psi^{(-)}_{Bb} | V^{(i)}_{Aa} - U_{Aa} | \chi^{(+)}_{Aa} \phi_A \phi_a \rangle$$ (prior form)

$$T_{fi} = \langle \chi^{(-)}_{Bb} \phi_B \phi_b | V^{(f)}_{Bb} - U_{Bb} | \Psi^{(+)}_{Aa} \rangle$$ (post form)

still exact expressions!

DWBA



distorted-wave Born approximation

- replace exact scattering wave functions

$\Psi^{(+)}_{Aa} \to \chi^{(+)}_{Aa}\phi_A\phi_a$ or $\Psi^{(-)}_{Bb} \to \chi^{(-)}_{Bb}\phi_B\phi_b$

- define overlap functions

$\Phi^a_{bx}(\vec{r}_{bx}) = \langle\phi_b|\phi_a\rangle$ and $\Phi^B_{Ax}(\vec{r}_{Ax}) = \langle\phi_A|\phi_B\rangle$

$\hat{=}$ wave function of transferred particle x

$\Rightarrow$ spectroscopic factors $\mathcal{S}^a_{bx} = \langle\Phi^a_{bx}|\Phi^a_{bx}\rangle$, $\mathcal{S}^B_{Ax} = \langle\Phi^B_{Ax}|\Phi^B_{Ax}\rangle$

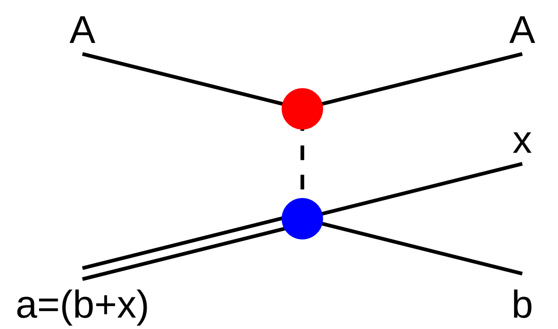
- convenient expressions for T-matrix elements

$T_{fi} \approx \langle\chi^{(-)}_{Bb}\Phi^B_{Ax}|V^{(i)}_{Aa} - U_{Aa}|\chi^{(+)}_{Aa}\Phi^a_{bx}\rangle$ (prior form)

$T_{fi} \approx \langle\chi^{(-)}_{Bb}\Phi^B_{Ax}|V^{(f)}_{Bb} - U_{Bb}|\chi^{(+)}_{Aa}\Phi^a_{bx}\rangle$ (post form)

only difference: potentials

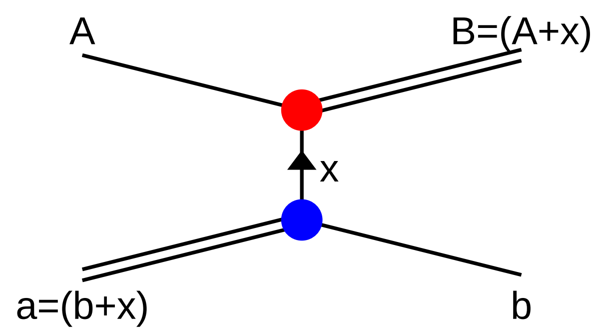
T-Matrix Elements for Direct Reactions



Coulomb Dissociation: direct breakup reaction

prior-form $$T_{fi} = \langle \Psi_f^{(-)} | V_{Aa}^{(i)} - U_{Aa} | \phi_A \phi_a \chi_{Aa}^{(+)} \rangle$$

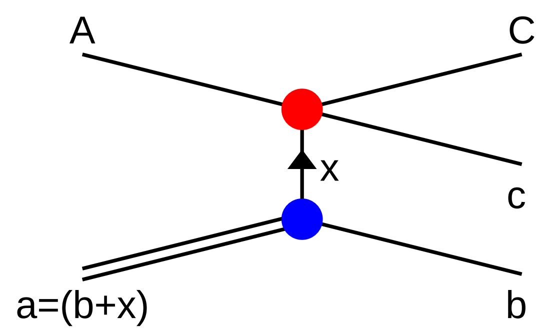
with bound-state wave function ϕ_a



ANC Method: transfer reaction to bound state

post-form $$T_{fi} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb}^{(f)} - U_{Bb} | \Psi_i^{(+)} \rangle$$

with bound-state wave function ϕ_B



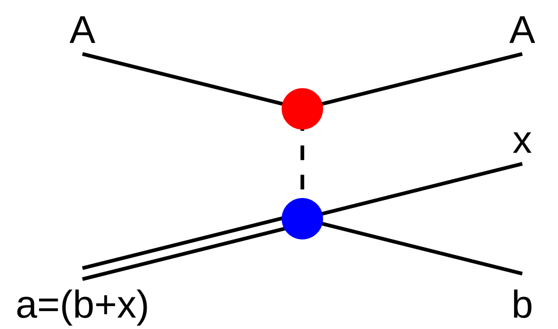
Trojan-Horse Method: transfer reaction to continuum

post-form $$T_{fi} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb}^{(f)} - U_{Bb} | \Psi_i^{(+)} \rangle$$

with scattering wave function $\phi_B = \Psi_{Cc}^{(-)}$

with exact scattering wave functions $\Psi_i^{(+)}, \Psi_f^{(+)}$, distorted waves $\chi_{Aa}^{(+)}, \chi_{Bb}^{(-)}$, full interactions $V_{Aa}^{(i)}, V_{Bb}^{(f)}$, and optical potentials U_{Aa}, U_{Bb}

Coulomb Dissociation I



- prior-form distorted-wave Born approximation (DWBA)

$$T_{fi} = \langle \chi^{(-)}_{A(bx)} \phi_A \Psi^{(-)}_{bx} | V^{(i)}_{Aa} - U_{Aa} | \phi_A \phi_a \chi^{(+)}_{Aa} \rangle$$

- neglection of nuclear interaction

- multipole expansion of Coulomb potential in far-field approximation ($r_{bx} > r_{Aa}$)

$$V^{(i)}_{Aa} - U_{Aa} = \frac{Z_A Z_b e^2}{|\vec{r}_b - \vec{r}_A|} + \frac{Z_A Z_x e^2}{|\vec{r}_x - \vec{r}_A|} - \frac{Z_A Z_a e^2}{|\vec{r}_a - \vec{r}_A|} \approx 4\pi Z_A e \sum_{\lambda\mu} \frac{Z^{(\lambda)}_{\mathrm{eff}} e}{2\lambda+1} \frac{r^{\lambda}_{bx}}{r^{\lambda+1}_{Aa}} Y_{\lambda\mu}(\hat{r}_{bx}) Y^{*}_{\lambda\mu}(\hat{r}_{Aa})$$

with effective charge numbers

$$Z^{(\lambda)}_{\mathrm{eff}} = Z_b \left(\frac{m_x}{m_b + m_x} \right)^{\lambda} + Z_x \left(- \frac{m_b}{m_b + m_x} \right)^{\lambda}$$

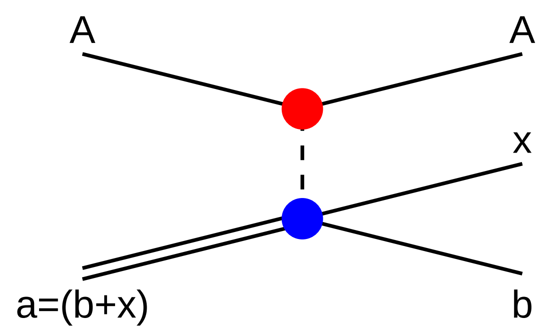
and relative coordinates $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, $\vec{r}_{Aa} = \vec{r}_A - \vec{r}_a$

$\Rightarrow$ factorization of T-matrix element

$$T_{fi} \approx 4\pi \sum_{\lambda\mu} \langle \Psi^{(-)}_{bx} | \underbrace{Z^{(\lambda)}_{\mathrm{eff}} e r^{\lambda}_{bx} Y_{\lambda\mu}(\hat{r}_{bx})} | \phi_a \rangle \langle \chi^{(-)}_{A(bx)} | \frac{Z_A e}{2\lambda+1} r^{-\lambda-1}_{Aa} Y^{*}_{\lambda\mu}(\hat{r}_{Aa}) | \chi^{(+)}_{Aa} \rangle$$

$\mathcal{M}(E\lambda\mu)$ electric multipole transition operator

Coulomb Dissociation II



⇒ **cross section** of Coulomb dissociation reaction

$$\frac{d^2\sigma}{dE_{bx}d\Omega_{Aa}} = \frac{1}{E_\gamma}\sum_{\pi\lambda}\sigma_{\pi\lambda}(a+\gamma\to b+x)\frac{dn_{\pi\lambda}}{d\Omega_{Aa}}$$

- virtual photon number $\frac{dn_{\pi\lambda}}{d\Omega_{Aa}}$ in quantal calculation or semiclassical approximation

⇒ $E2$ enhancement $\frac{dn_{E2}}{d\Omega_{Aa}}\Big/\frac{dn_{E1}}{d\Omega_{Aa}} \approx \frac{4\hbar^2c^2}{E_\gamma^2 b^2}$, $M1$ suppression $\frac{dn_{M1}}{d\Omega_{Aa}}\Big/\frac{dn_{E1}}{d\Omega_{Aa}} \approx \frac{v^2}{c^2}$

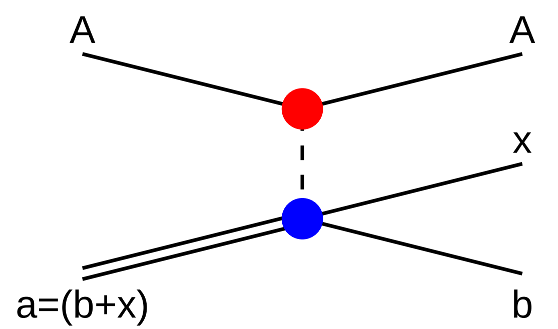
- theorem of detailed balance

$$\sigma_{\pi\lambda}(b+x\to a+\gamma) = \frac{2(2J_a+1)}{(2J_b+1)(2J_x+1)}\frac{k_\gamma^2}{k_{bx}^2}\sigma_{\pi\lambda}(a+\gamma\to b+x)$$

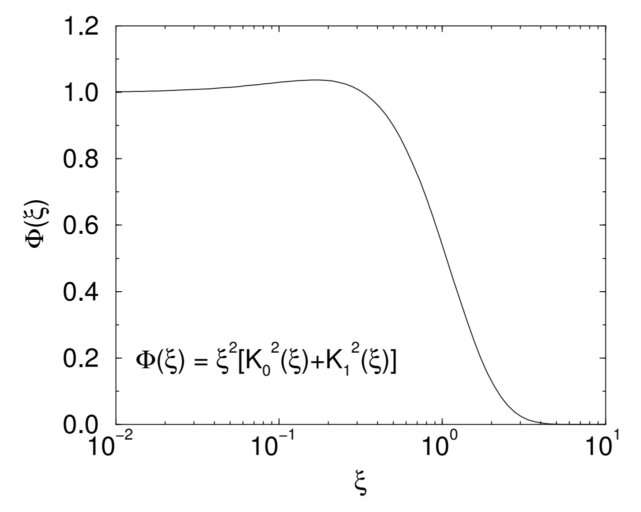
with radiative capture and photo dissociation cross sections

- phase space factor $\frac{k_\gamma^2}{k_{bx}^2} = \frac{(E_{bx}+S_{bx})^2}{2\mu_{bx}c^2E_{bx}} \ll 1$ for (not too) small E_{bc}

Coulomb Dissociation III



virtual photon spectrum (E1)



(Fermi 1924, Weizsäcker-Williams 1932)

characteristic parameters

- adiabaticity parameter

$$\xi = \frac{\omega b}{\gamma v}$$

$\hbar\omega$	excitation energy
b	impact parameter
v	projectile velocity

$\xi = 0$: sudden excitation

$\xi \gg 1$: adiabatic excitation

$\xi \approx 1 \Rightarrow E_{\mathrm{exc}}^{\mathrm{max}} \approx \gamma v \hbar / b$

- strength parameter

$$\chi = \frac{Z_A e \langle f || \mathcal{M}(\pi\lambda) || i \rangle}{\hbar v b^{\lambda}}$$

$Z_A e$	target charge
$\mathcal{M}(\pi\lambda)$	multipole operator

χ small $\Rightarrow$ first-order perturbation theory sufficient

χ large $\Rightarrow$ higher-order effects

^{7}Be(p,γ)^{8}B

nuclear astrophysics:

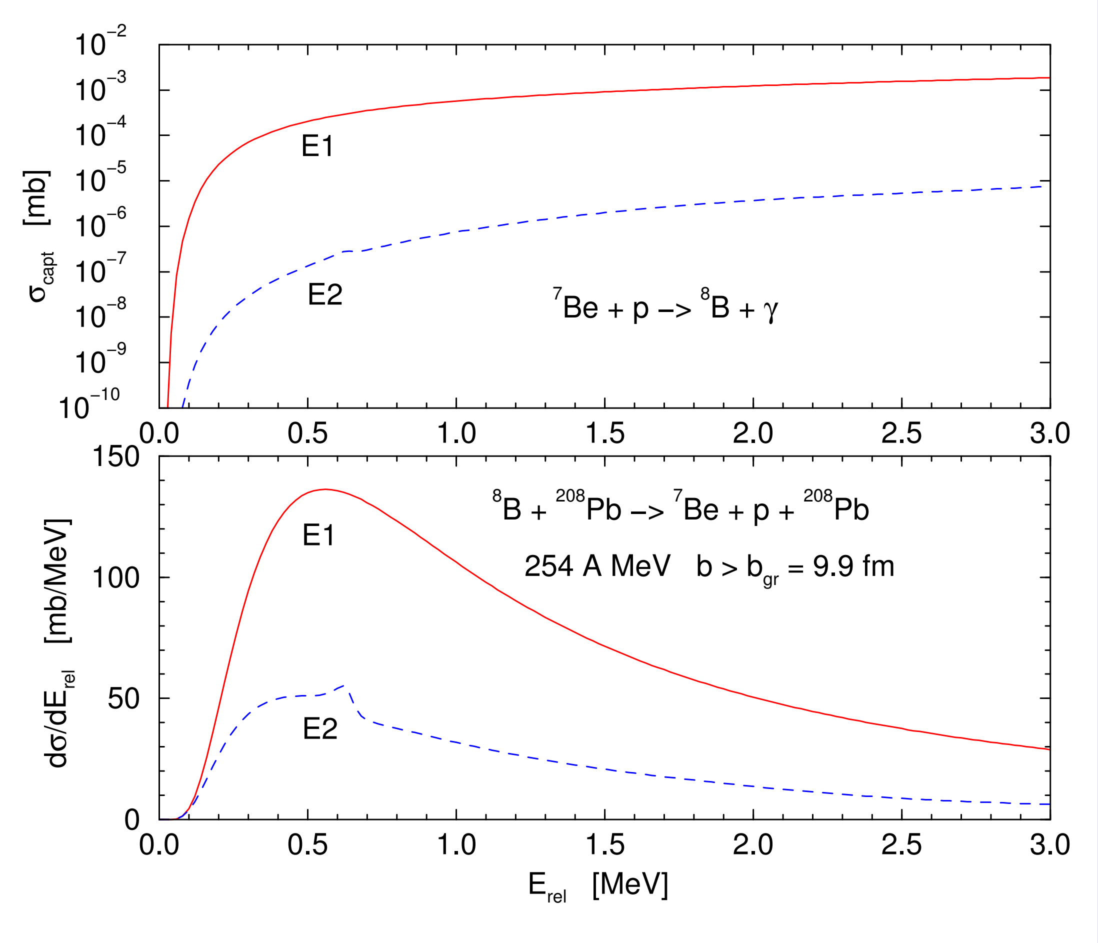
- small branch of pp chain

  . . . ^{7}Be$(p,\gamma)^8$B$(e^+\nu_e)^8$Be $\longrightarrow$ 2^{4}He

  $\Rightarrow$ source of high-energy neutrinos
- $E_{\mathrm{eff}} \approx 20$ keV for ^{7}Be$(p,\gamma)^8$B in sun
- neutrino flux $\propto$ S factor S_{17}

calculation in potential model

- capture cross section at low energies dominated by non-resonant E1 transition to p-wave ground state with 137 keV binding energy
- enhancement of E2 contribution in Coulomb dissociation reaction, relevant?



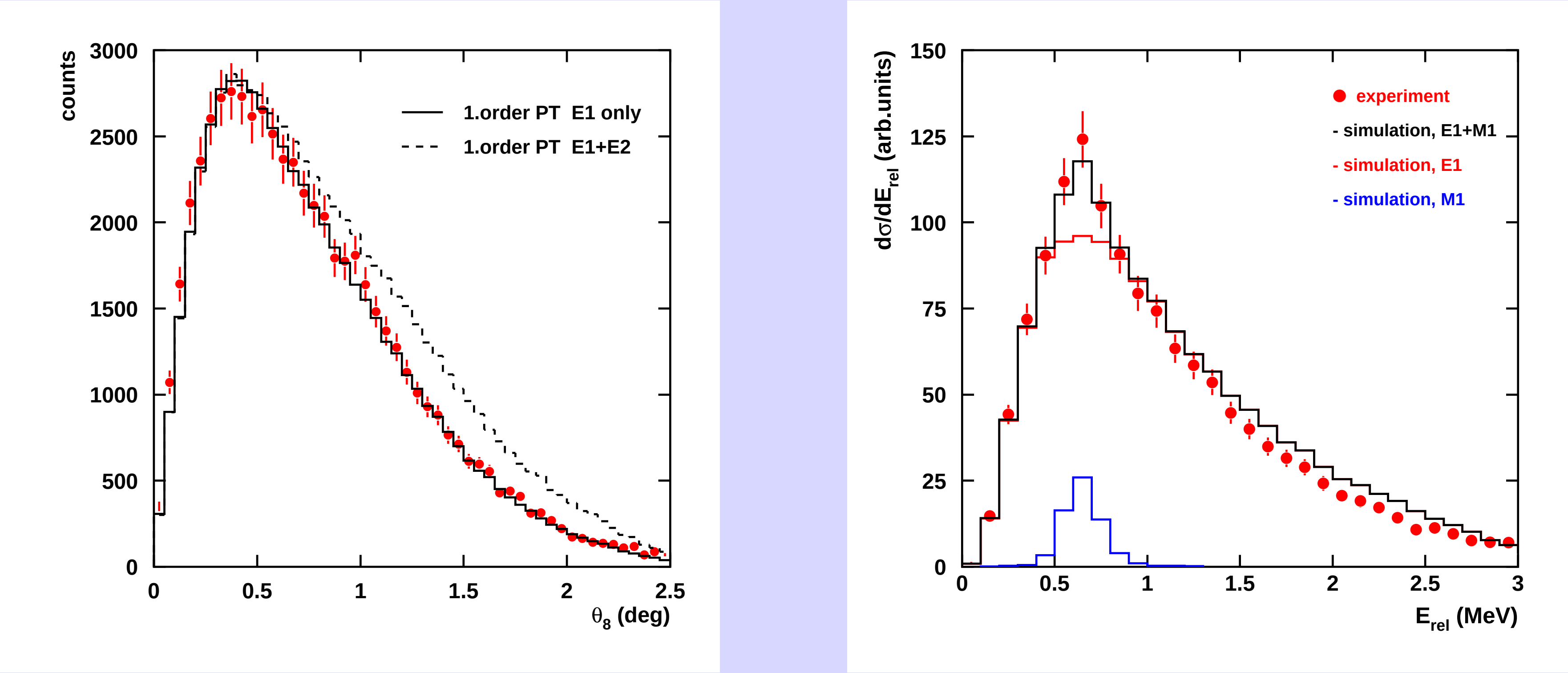
(M1 contribution of sharp resonance at 632 keV not shown)

^{7}Be(p,γ)^{8}B

- **GSI experiment:** Coulomb breakup with 254 A MeV ^{8}B beam on a Pb target
  F. Schümann, S. Typel, F. Hammache et al., Phys. Rev. C 73 (2006) 015806

  Monte Carlo simulation of distributions in semiclassical first-order perturbation theory

  (a) scattering angle of excited ^{8}B (b) ^{7}Be-p relative energy



$\Rightarrow$ E2 contribution very small (also found in other angular distributions)

^{7}Be(p,γ)^{8}B

recent direct experiments (^{7}Be target)

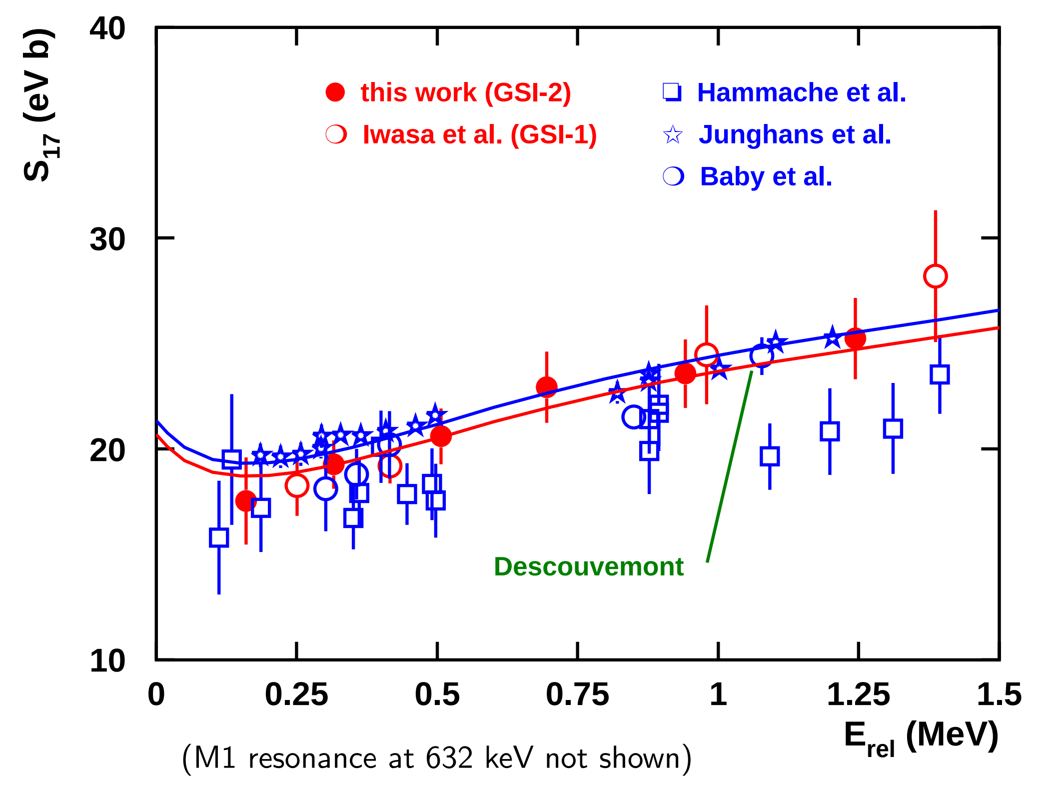
- Orsay: F. Hammache et al.,
  Phys. Rev. Lett. 80 (1998) 928; 86 (2001) 3985
- University of Washington, Seattle: A.R. Junghans et al.,
  Phys. Rev. C 68 (2003) 065803
- Weizmann Institute, Rehovot: L.T. Baby et al.,
  Phys. Rev. C 67 (2003) 065805

Coulomb breakup experiments (Pb target)

- RIKEN: 46.5 A MeV/51.2 A MeV
  T. Motobayashi et al., Phys. Rev. Lett. 73 (1994) 2680
  T. Kikuchi et al., Eur. Phys. J. A3 (1998) 213
- MSU: 83 A MeV
  B. Davids et al., Phys. Rev. C 63 (2001) 065806
- GSI: 254 A MeV
  N. Iwasa et al., Phys. Rev. Lett. 83 (1999) 2910
  F. Schümann et al., Phys. Rev. Lett. 90 (2003) 232501
  Phys. Rev. C 73 (2006) 015806

theoretical model for extrapolation to E=0 MeV

- P. Descouvement, Phys. Rev. C 70 (2004) 065802



(M1 resonance at 632 keV not shown)

- Seattle: $S_{17}(0) = 21.2 \pm 0.5$ eV b
- GSI-2: $S_{17}(0) = 20.6 \pm 0.8(\text{stat.}) \pm 1.2(\text{syst.})$ eV b

⇒ consistent results of different methods

Coulomb Dissociation IV

- contribution of **nuclear interaction** ⇒ absorption, breakup
  - quantal calculations (DWBA/Eikonal), optical potentials needed

- **higher-order effects** of Coulomb interaction $\hat{=}$ multi-photon exchange
  ⇒ post-acceleration of fragments after breakup
  - various theoretical approaches:
    - ○ higher-order perturbation theory / time-dependent dynamical calculations
    - ○ full quantal approaches using three-body wave function with appropriate asymptotics
    - ○ analytical result in certain limits

- consistency of semiclassical and quantal three-body calculations?
  E. O. Alt et al., Phys. Rev. C 71 (2005) 024605

- alternative analysis via ANC: consistent with CD results for $S_{17}(0)$
  K. Ogata et al., Phys. Rev. C 73 (2006) 024605

⇒ compare various theoretical approaches for the same experimental conditions

⇒ select appropriate range of scattering angles and projectile energy

Reduced Transition Probability and Matrix Elements

- **photo dissociation cross section** for electric transition

$$\sigma_{E\lambda}(a+\gamma \to b+x) = \frac{\lambda+1}{\lambda}\frac{(2\pi)^3}{[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE}$$

  with photon energy $E_\gamma = S_{bx} + E$, $E = \hbar^2 k^2/(2\mu_{bx})$

- **reduced transition probability**

$$\frac{dB}{dE}(E\lambda) = \frac{2J_{bx}+1}{2J_a+1}\left|\langle kJ_{bx}||\mathcal{M}(E\lambda)||J_a\rangle\right|^2\frac{\mu_{bx}k}{(2\pi)^3\hbar^2}$$

  with electric multipole operator $\mathcal{M}(E\lambda\mu)$

- **reduced matrix element** (neglecting spins)

$$\langle kl_f||\mathcal{M}(E\lambda)||l_i\rangle = \frac{4\pi}{k}Z_{\text{eff}}^{(\lambda)}eD_{l_i}^{l_f}(\lambda)(-i)^{l_f}I_{l_i}^{l_f}(\lambda)$$

  for $E\lambda$ transition with angular momentum coupling coefficient $D_{l_i}^{l_f}(\lambda)$ and

  radial integral $I_{l_i}^{l_f}(\lambda) = \int_0^\infty dr\, g_{l_f}^*(r)\, r^\lambda\, f_{l_i}(r)$ with radial wave functions

  f_{l_i} and g_{l_f} of bound and scattering state, respectively

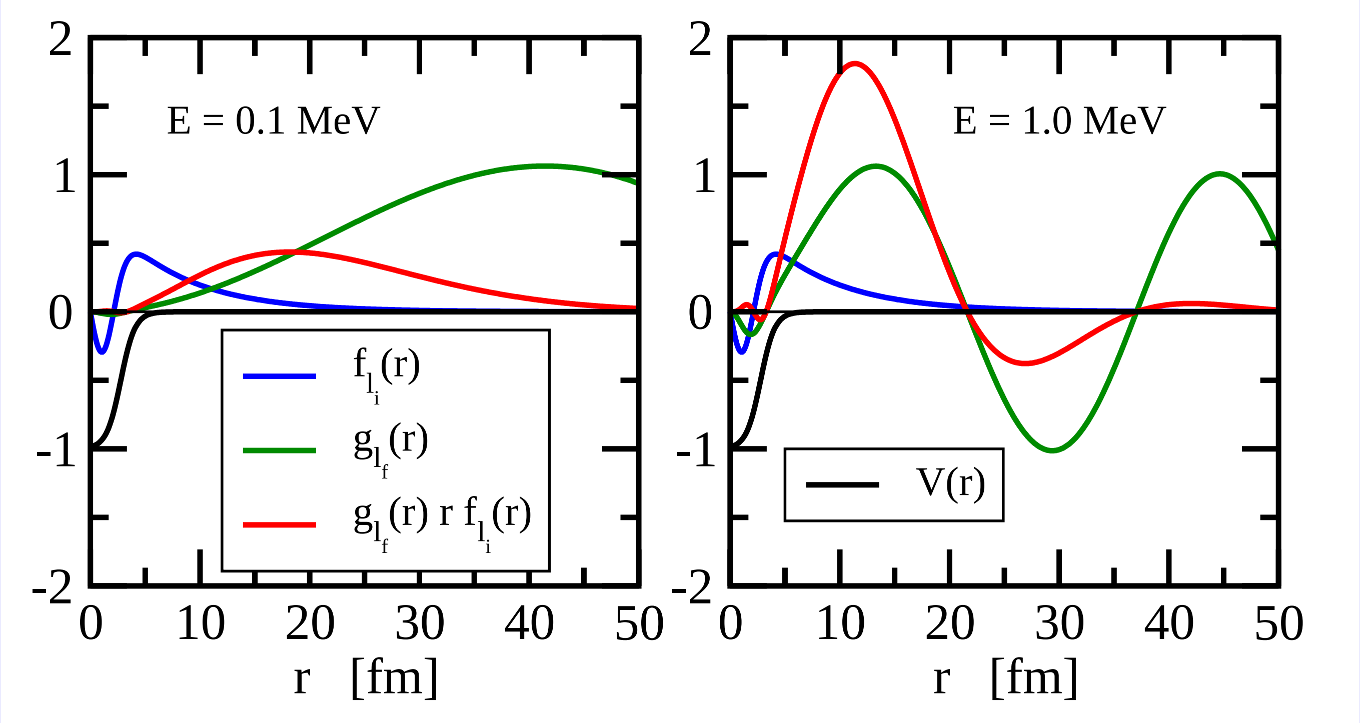
Radial Integral

- **example**: breakup of ^{11}Be $\rightarrow$ ^{10}Be $+$ n

  neutron halo nucleus with neutron separation energy $S_n = 0.504$ MeV

  $E1$ transition from s wave bound state to p wave scattering state with energy E

  $\Rightarrow$ integrand in radial integral



- Eλ transitions at low relative energies

  $\Rightarrow$ matrix elements determined by asymptotic of wave functions

Asymptotics of Wave Functions

- **bound state** wave function $\phi_a(lm) = \frac{1}{r} f_l(r) Y_{lm}(\hat{r}) \phi_b \phi_x$ (neglecting spins)

  with radial wave function $f_l(r) \to C^a_{bx}(l) W_{-\eta_a, l+1/2}(2qr)$

  asymptotic normalization coefficient (ANC) $C^a_{bx}(l)$, Whittaker function $W_{-\eta_a, l+1/2}$

  separation energy $S_{bx} = \hbar^2 q^2/(2\mu_{bx})$, Sommerfeld parameter $\eta_a = Z_b Z_x e^2 \mu_{bx}/(\hbar^2 q)$

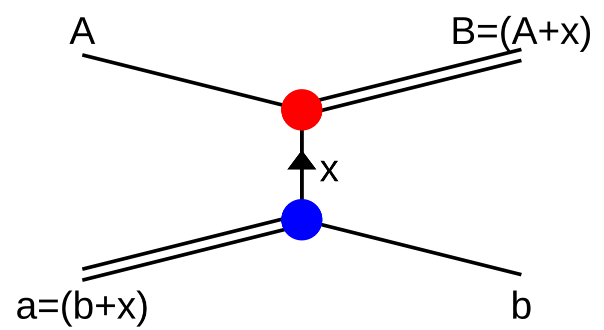
- elastic **scattering** wave function $\chi^{(+)}_{bx}(lm) = \frac{4\pi}{kr} g^{(+)}_l(r) i^l Y^*_{lm}(\hat{k}) Y_{lm}(\hat{r}) \phi_b \phi_x$

  with radial wave function $g^{(+)}_l(r) \to \frac{1}{2i} \left[S^l_{bx} u^{(+)}_l(\eta_{bx}; kr) - u^{(-)}_l(\eta_{bx}; kr) \right]$

  S matrix element $S^l_{bx} = \exp\left[2i\left(\delta_l + \sigma_l\right)\right]$ with nuclear phase shift δ_l,

  Coulomb phase shift σ_l, and wave functions $u^{(\pm)}_l = \exp(\mp i \sigma_l) \left[G_l \pm i F_l \right]$
  relative energy $E = \hbar^2 k^2/(2\mu_{bx})$, Sommerfeld parameter $\eta_{bx} = Z_b Z_x e^2 \mu_{bx}/(\hbar^2 k)$

ANC Method



- distorted-wave Born approximation (DWBA)
- replace exact overlap functions by asymptotic form with asymptotic normalization coefficients (ANCs) and Whittaker functions

⇒ **overlap functions** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$$\langle \phi_b | \phi_a \rangle \approx \frac{C^a_{bx}}{r_{bx}} W_{-\eta_{bx}, l_a+1/2}(2q_{bx}r_{bx}) Y_{l_a m_a}(\hat{r}_{bx}) \phi_x$$ and $$\langle \phi_A | \phi_B \rangle \approx \dots$$

⇒ **cross section** of transfer reaction to bound state

$$\frac{d\sigma}{d\Omega_{Bb}} = \left|C^a_{bx}\right|^2 \left|C^B_{Ax}\right|^2 \frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$ with reduced DWBA cross section $$\frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$

⇒ calculate S factor at zero energy of capture reaction $b(x,\gamma)a$ numerically

- $\left|C^a_{bx}\right|^2 \Leftrightarrow S(0)$ unique relation? effect of $b-x$ interaction?
- precise optical potentials and one additional ANC needed

^{7}Be$(p,\gamma)^{8}$B

experiments: (Texas A&M University)

extraction of ANC from

- proton transfer reactions
  ^{10}B(^{7}Be,^{8}B)^{9}Be, ^{14}N(^{7}Be,^{8}B)^{13}C
  with 85 MeV ^{7}Be beam

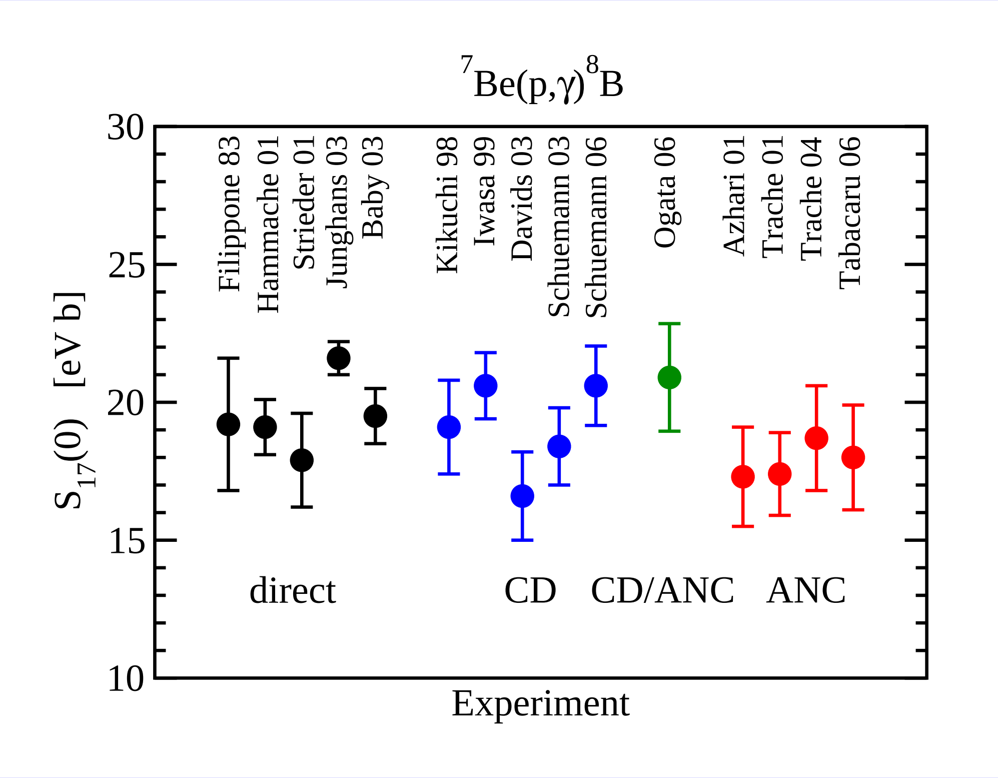
  A. Azhari et al., PRC 63 (2001) 055803

  G. Tabacaru et al., PRC 73 (2006) 025808
- breakup ^{8}B → ^{7}Be+p
  on C, Si, Sn, and Pb targets with beam energies from 30 to 1000 A MeV

  L. Trache et al., PRL 87 (2001) 271102,
  PRC 69 (2004) 032802
- neutron transfer reaction
  ^{13}C(^{7}Li,^{8}Li)^{12}C with 63 MeV ^{7}Li beam and charge symmetry

  L. Trache et al., PRC 67 (2003) 062801(R)

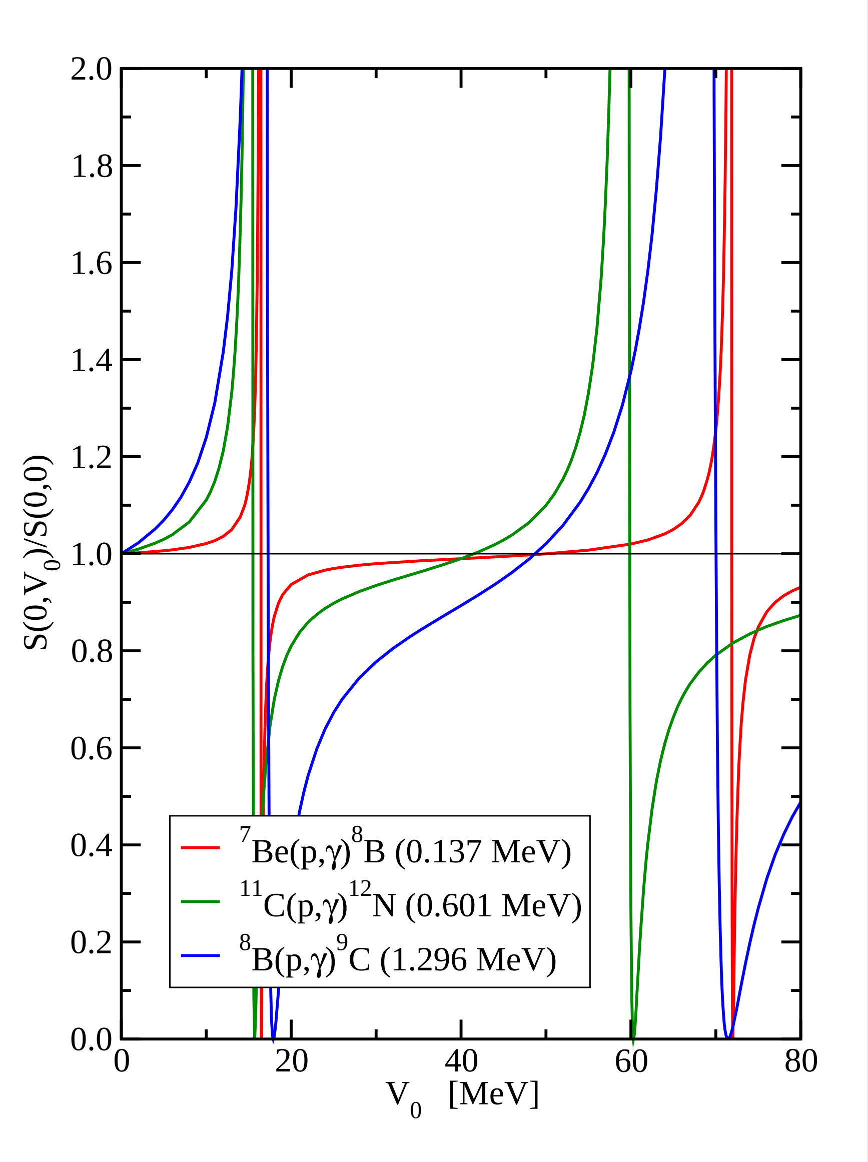
comparison with other methods



Zero-Energy S Factor and Interaction in Continuum

- effects of interaction in continuum states
  - modification of shape of cross section, S factor (i.e. energy dependence)
  - change of $S(0)$ even though $\delta \to 0$
- calculation of zero-energy S factor $S(0)$ in single-particle model with Woods-Saxon potential with different depths V_0
- example: $E1\ s \to p$ wave capture for different nuclei with proton+core structure
  $\Rightarrow$ stronger variation of $S(0)$ with V_0 with larger proton separation energy
- simple relation ANC $\Leftrightarrow S(0)$ only correct for halo nuclei

S. Typel and G. Baur, Nucl. Phys. A 759 (2005) 245



Coulomb Dissociation of ^{11}Be

- $E1$ transition from s-wave halo ground state ($S_n = 504$ keV) to p-wave continuum states with $j = 3/2, 1/2$
- effective-range expansion for phase shifts

$$k^{2l+1} \cot \delta_l^j = -1/a_l^j + \ldots$$

with scattering length a_l^j

- fit to experimental data from Coulomb breakup of ^{11}Be at 520 A·MeV on Pb (R. Palit et al., PRC 68 (2003) 034318)

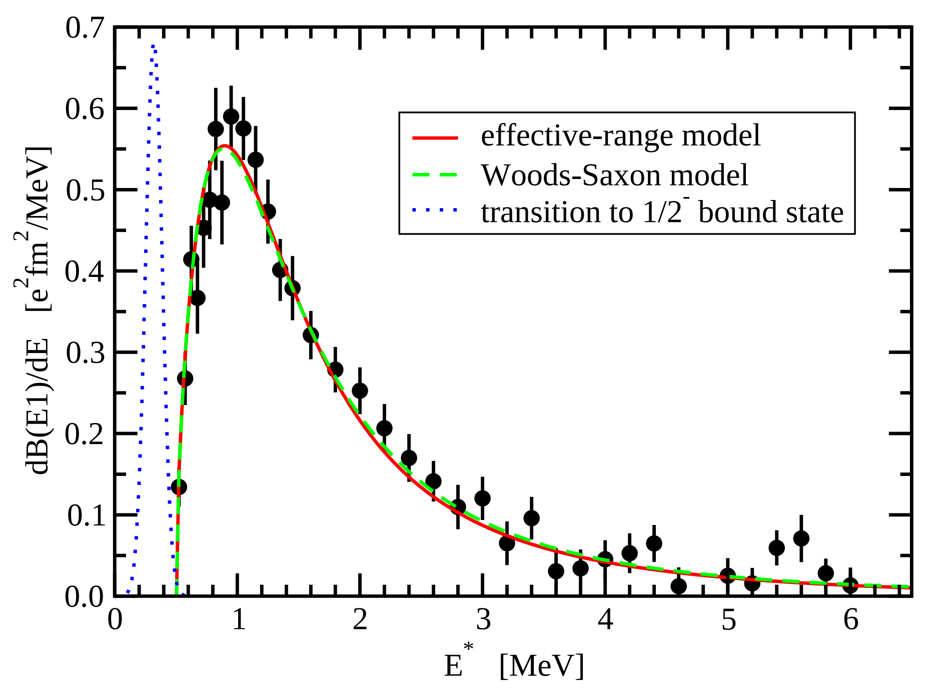
$\Rightarrow$ ANC $C_0 = 0.724(8)$ fm$^{-1/2}$

$\Rightarrow$ spectroscopic factor $C^2S = 0.704(15)$

$\Rightarrow$ scattering lengths

$$a_1^{3/2} = -1.48(3.44, -3.40) \text{ fm}^3$$
$$a_1^{1/2} = 456.6(67.4, -65.8) \text{ fm}^3$$

S. Typel and G. Baur, PRL 93 (2004) 142502

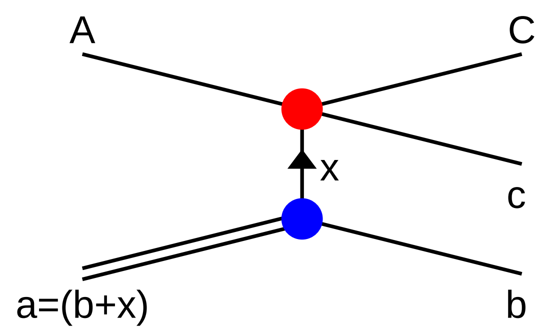


- $a_1^{1/2}$ unnaturally large

$\Leftrightarrow$ existence of bound $1/2^-$ state 320 keV above ground state

$\Rightarrow$ reduced $E1$ strength in continuum

Trojan-Horse Method I



- distorted-wave Born approximation (DWBA)
- use asymptotic form of scattering wave function in reaction channel $C + c \to A + x$ (essential "surface approximation")

⇒ **overlap function** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$$\langle \phi_A | \Psi^{(-)}_{Cc} \rangle \approx \frac{4\pi}{k_{Cc} r_{Ax}} \sqrt{\frac{v_{Cc}}{v_{Ax}}} \sum_{lm} \xi_l^*(r_{Ax}) i^l Y_{lm}(\hat{r}_{Ax}) Y^*_{lm}(\hat{k}_{Cc}) \phi_x$$

with $\xi_l(r_{Ax}) = \frac{1}{2i} \left[S^l_{AxCc} u_l^{(+)}(\eta_{Ax}; k_{Ax} r_{Ax}) - \delta_{AxCc} u_l^{(-)}(\eta_{Ax}; k_{Ax} r_{Ax}) \right]$

and S-matrix element S^l_{AxCc} of reaction $C(c,x)A$

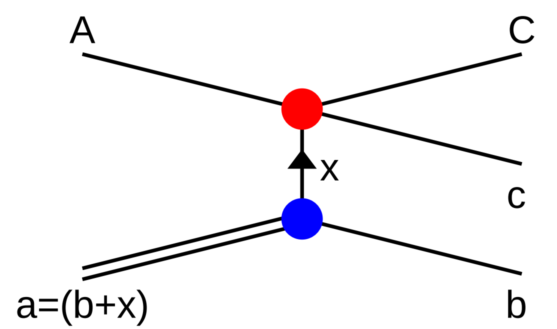
⇒ **cross section** of transfer reaction to continuum (single channel, $Ax \neq Cc$)

$$\frac{d^3\sigma}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}} = \left| S^l_{AxCc} \right|^2 \frac{d^3\tilde{\sigma}_l}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}}$$

with reduced DWBA cross section

general theory: S. Typel and G. Baur, Ann. Phys. (N.Y.) 305 (2003) 228

Trojan-Horse Method II



additional approximations
(not necessary in general, but convenient)

- potential $V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi^{(-)}_{Bb}$, $\chi^{(+)}_{Aa}$

⇒ **cross section** of transfer reaction to continuum (single channel)

$$\boxed{\frac{d^3\sigma}{d\Omega_{Bb}d\Omega_{Cc}dE_{Cc}} = K\, W(\vec{Q}_{Bb}) \frac{d\sigma_l}{d\Omega}(Ax \to Cc)\, T_l(k_{Ax})}$$

with kinematic factor K

- **momentum distribution** $W(\vec{Q}_{Bb}) = |\tilde{\Phi}^{a}_{bx}(\vec{Q}_{Bb})|^2$

  depending on momentum transfer to spectator b ⇒ quasi-free scattering conditions

- **cross section** $\frac{d\sigma_l}{d\Omega}(Ax \to Cc)$ of two-body reaction

- **penetration factor** $T_l(k_{Ax}) \approx k^3_{Ax} \exp(2\pi\eta_{Ax})$

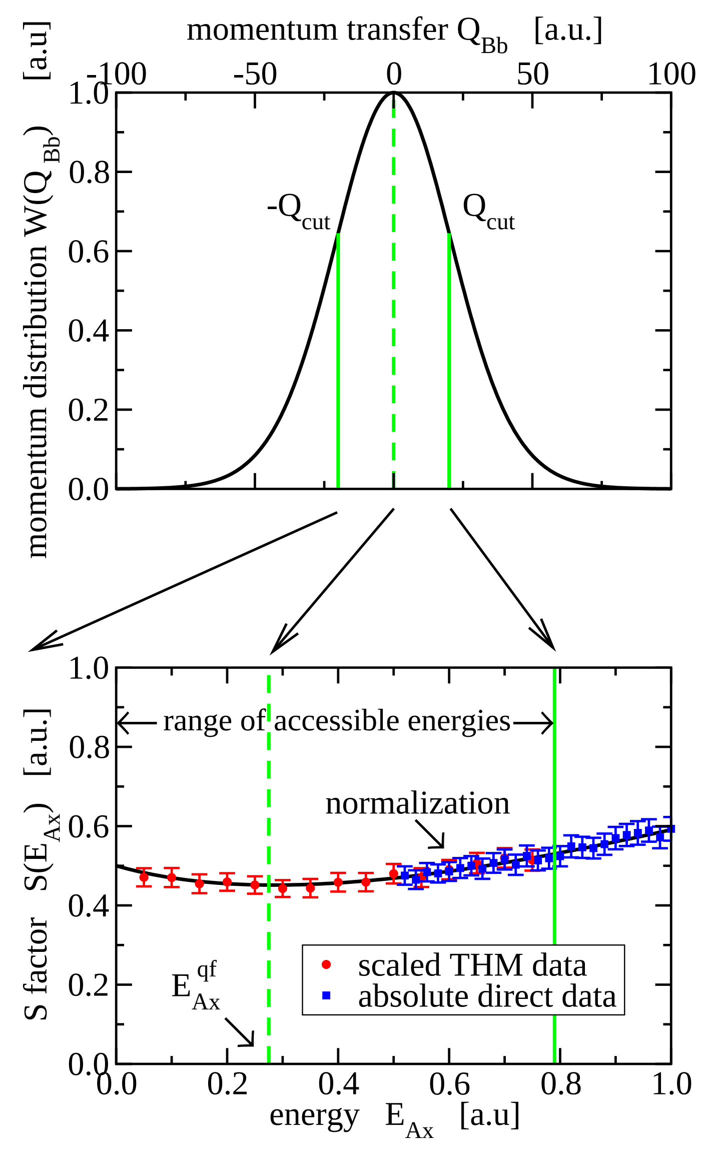
⇒ cancels suppression of two-body cross section by Coulomb barrier for $E_{Ax} \to 0$

Application of the Trojan-Horse Method

- selection of Trojan horse $a = b + x$
  (e.g. ^{2}H $= n + p$, ^{6}Li $= \alpha + d$, $\ldots$)
  with binding energy $\epsilon_a > 0$ and
  well known ground state wave function
  $\Rightarrow$ momentum distribution $W(\vec{Q}_{Bb})$
- width of momentum distribution W
  $\Leftrightarrow$ Fermi motion of x inside a
- condition $\vec{Q}_{Bb} = 0$ defines
  "quasi-free energy" in $A + x$ system

$$E_{Ax}^{qf} = E_{Aa}\left(1 - \frac{\mu_{Aa}}{\mu_{Bb}}\frac{\mu_{bx}^2}{m_x^2}\right) - \epsilon_a \ll E_{Aa}$$

- cutoff in $\vec{Q}_{Bb}$ determines range of
  accessible energies E_{Ax} around E_{Ax}^{qf}
- small momentum transfer
  $\Rightarrow$ dominance of quasi-free process
- normalization of cross section to direct data
  at higher E_{Ax}



^{2}H(^{6}Li,α)^{4}He

- **direct reaction**: ^{2}H(^{6}Li,α)^{4}He
  - ○ experiment with gas target

    (S. Engstler et al., Z. Phys. A 342 (1992) 471)
  - ○ $S(0) = 17.4$ MeV b

    (corrected for electron screening)

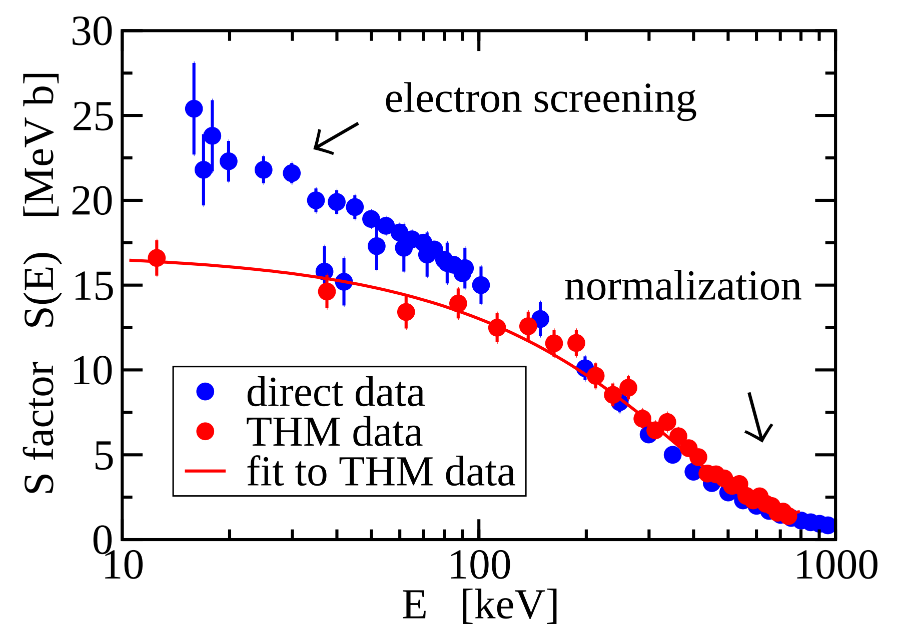
- **THM**: ^{6}Li(^{6}Li,$\alpha\alpha$)^{4}He
  - ○ experiment with 6 MeV ^{6}Li beam

    (C. Spitaleri et al., Phys. Rev. C 63 (2001) 055801;

    A. Musumarra et al., Phys. Rev. C 64 (2001) 068801)
  - ○ $E^{qf} = 25$ keV
  - ○ target and projectile breakup
  - ○ $l = 0$, $\hbar Q_{Bb} < 35$ MeV/c
  - ○ normalization to direct data

    for $E > 600$ keV

    $\Rightarrow S(0) = (16.9 \pm 0.5)$ MeV b



- **electron screening potential**:

  U_e(direct) $= (330 \pm 120)$ eV

  U_e(THM) $= (320 \pm 50)$ eV

  U_e(theory) $= 186$ eV (adiabatic limit)

^{6}Li(p,α)^{3}He

- **direct reaction**: ^{6}Li(p,α)^{3}He
  - ○ experimental data

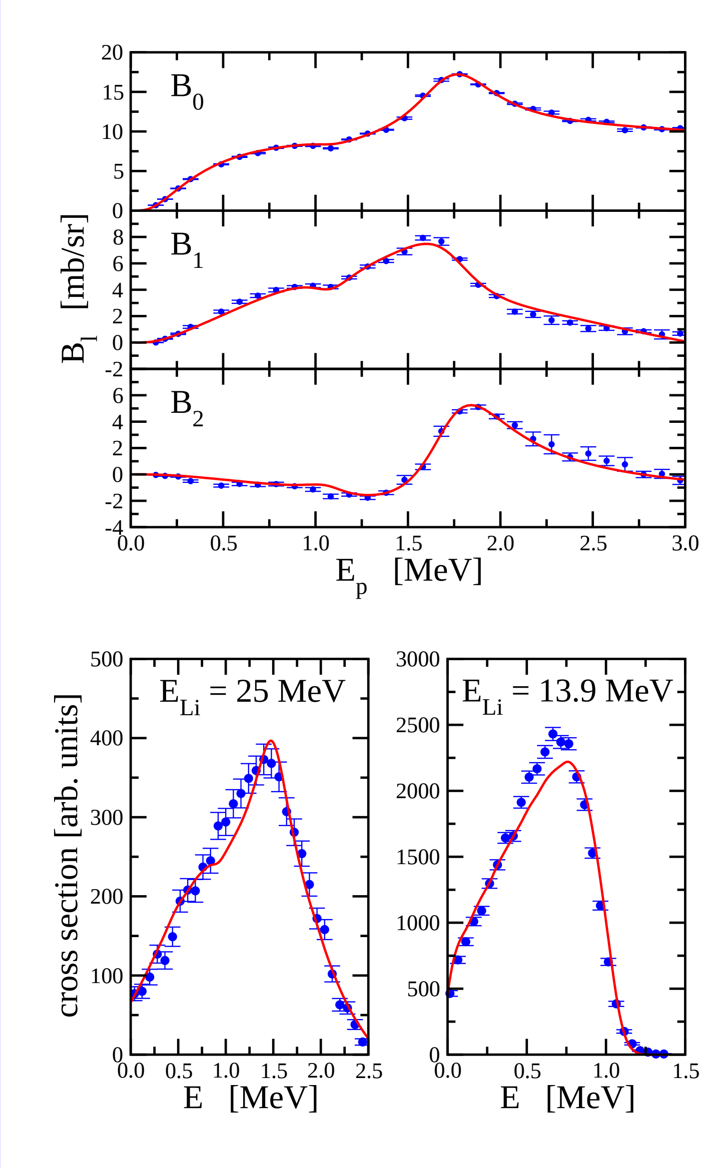
    (J. Elwyn et al., Phys. Rev. C 20 (1979) 1084)
  - ○ differential cross section

    $$d\sigma/d\Omega = \sum_l B_l P_l(\cos\theta)$$
  - ○ non-resonant s wave and
    resonant p wave contribution
  - ○ S matrix from R-matrix fit
    $\Rightarrow$ simulation of THM experiment

- **THM**: ^{2}H(^{6}Li,α^{3}He)n
  - ○ experiments with 13.9/25 MeV ^{6}Li beam

    (A. Tumino et al., Phys. Rev. C 67 (2003) 065803

    and preliminary results)
  - ○ $E^{qf} = -0.24/1.35$ MeV
  - ○ $\hbar Q_{Bb} < 30$ MeV/c
  - ○ finite cross section at $E = 0$ MeV!



Trojan-Horse Method III

- recent **applications**: rearrangement reactions

^{2}H(d,p)^{3}H	$\Rightarrow$	^{2}H(^{6}Li,p t)^{4}He	A. Tumino et al., Eur. J. Phys. A 25, 649 (2005)
^{3}He(d,p)^{4}He	$\Rightarrow$	^{6}Li(^{3}He,p α)^{4}He	M. La Cognata et al., Phys. Rev. C 71, 064301 (2005)
^{6}Li(p,α)^{3}He	$\Rightarrow$	^{2}H(^{6}Li,α ^{3}He)n	A. Tumino et al., Phys. Rev. C 67, 065803 (2003)
^{6}Li(n,t)^{4}He	$\Rightarrow$	^{2}H(^{6}Li,p t)^{4}He	A. Tumino et al., Eur. J. Phys. A 25, 649 (2005)
^{6}Li(d,α)^{4}He	$\Rightarrow$	^{6}Li(^{6}Li,$\alpha\alpha$)^{4}He	A. Musumarra et al., Phys. Rev. C 64, 068801 (2001)
^{7}Li(p,α)^{4}He	$\Rightarrow$	^{2}H(^{7}Li,$\alpha\alpha$)n	M. Lattuada et al., Astrophys. J. 562, 1076 (2001)
^{9}Be(p,α)^{6}Li	$\Rightarrow$	^{2}H(^{9}Be,^{6}Li α)n	C. Spitaleri et al., Proceedings, Hanoi (2004)
^{11}B(p,α)^{8}Be	$\Rightarrow$	^{2}H(^{11}B,α ^{8}Be)n	S. Romano et al., Nucl. Phys. A 738, 406 (2004)

analysis only in simple theoretical approximations
$\Rightarrow$ full DWBA calculations needed under quasi-free scattering conditions
with consistent treatment of bound/scattering/resonant states
(numerically very demanding)

- finite cross section at $E_{Ax} = 0 \Rightarrow$ continue to $E_{Ax} < 0$:
  investigation of subthreshold resonances

Trojan-Horse Method IV

- extension to radiative capture reactions possible
  ⇒ additional approach independent from Coulomb dissociation and ANC methods

- study elastic scattering without Coulomb contribution ⇒ optical potentials

- application to reactions with exotic nuclei ⇒ large cross sections

- extracted S factor not affected by electron screening
  ⇒ determination of electron screening potential U_e by comparison to direct data
  ⇒ consistent values for U_e, larger than adiabatic limit, challenge for theory

Conclusions

- **Indirect methods** provide complementary information on reactions of astrophysical interest
  - Coulomb dissociation method
  - method of asymptotic normalization coefficients (ANC)
  - Trojan-Horse method
- similar characteristics and theoretical concepts
- importance of nuclear reaction theory
  - direct reactions with certain kinematical conditions
  - peripheral reactions, asymptotics of wave functions
  - approximations ⇒ range of validity, accuracy
- great potential for future applications (nuclear astrophysics, structure and reactions of exotic nuclei, . . .)
- dedicated theoretical investigations needed in close collaboration with experiment