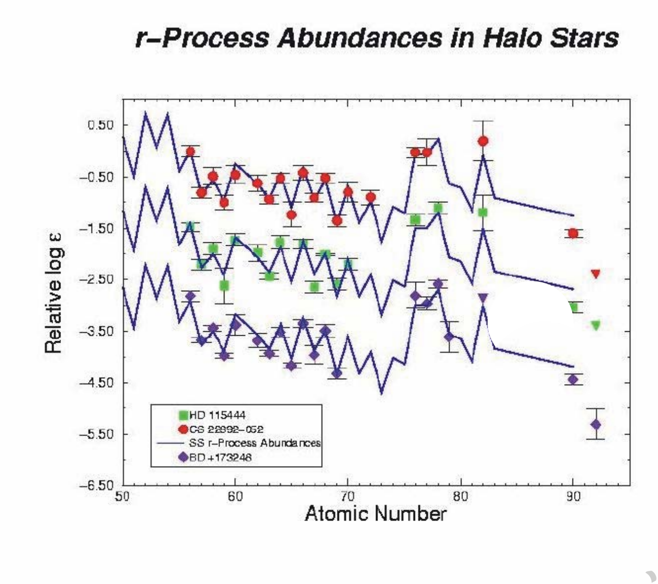
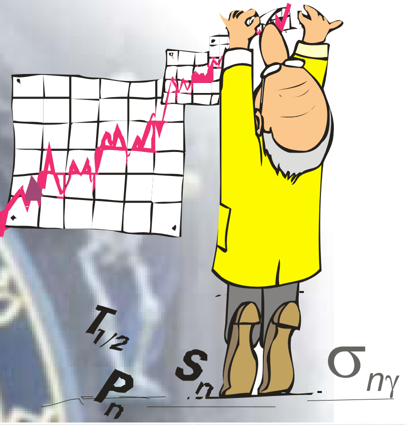
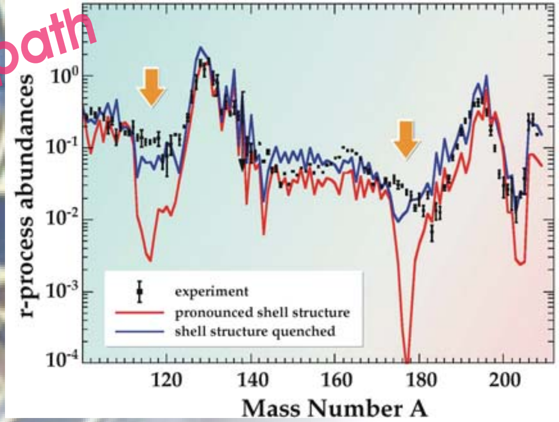
Gross β–decay properties and the A=130 $N_{r,\odot}$ peak





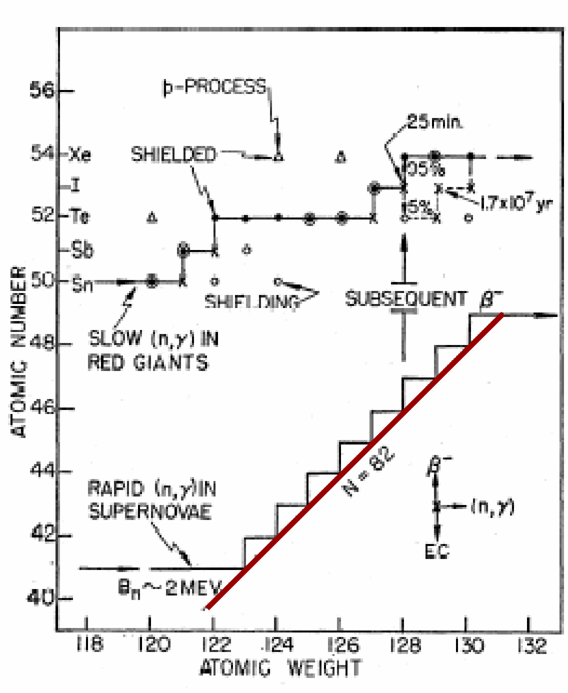


Karl-Ludwig Kratz

- Institut für Kernchemie, Univ. Mainz, Germany
- HGF VISTARS, Germany
- Department of Physics, Univ. of Notre Dame, USA

The gross β–decay quantity $T_{1/2}$ and the A ≈ 130 $N_{r,\odot}$ peak

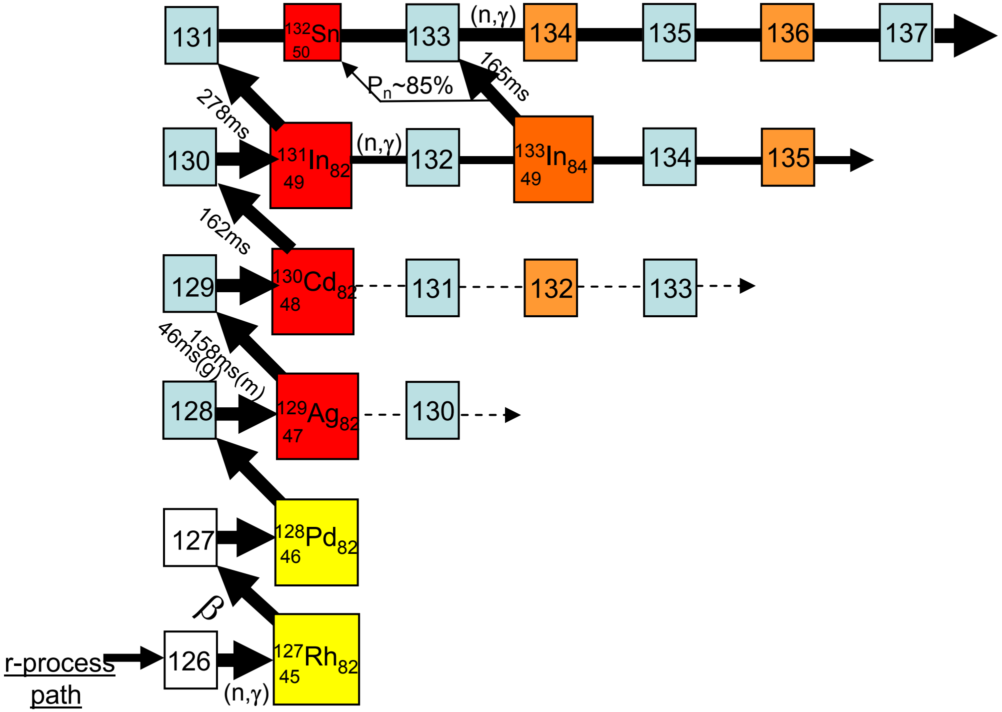
already B²FH (Revs. Mod. Phys. 29; 1957)
C.D. Coryell (J. Chem. Educ. 38; 1961)



"climb up the staircase" at N=82;
major waiting point nuclei;
"break-through pair" ^{131}In, ^{133}In;

↷ "association with the rising side of major peaks in the abundance curve"

...still today important r-process properties to be studied experimentally and theoretically.

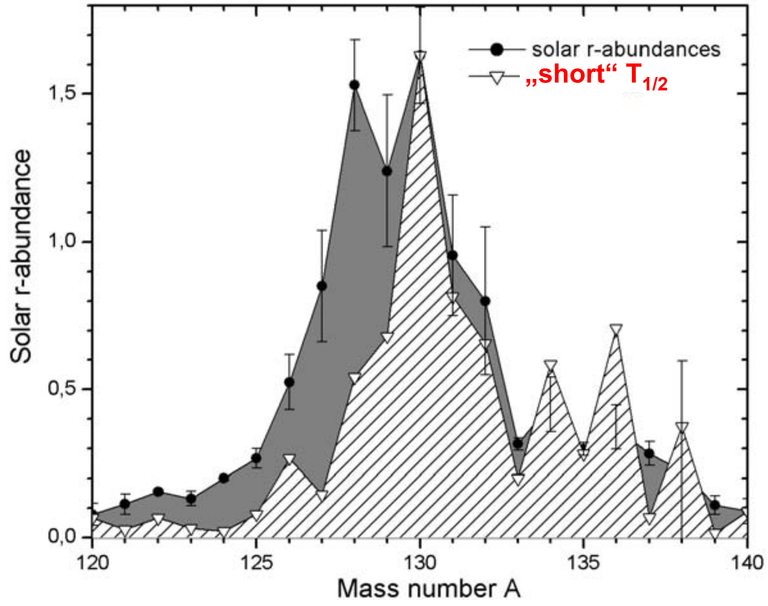


K.-L. Kratz (Revs. Mod. Astr. 1; 1988)

climb up the N= 82 ladder ...
A ≈ 130 "bottle neck"
$T_{1/2}$ ⇨ **total r-process duration τ_r**

The gross β-decay quantity $T_{1/2}$ and the shape of $N_{r,\odot}$ peak at A=130

Calculated abundances with „waiting-point“ concept, normalized to ^{130}Te according to experimental $T_{1/2}(^{130}\mathrm{Cd})$.



Deficiencies at rising wing 122<A<130

- neutrino induced reactions ?
  Qian,Haxton et al. (1997)
- waiting-point concept breaks down ?
  Martinez-P. & Langanke (1999)
- nuclear structure below ^{132}Sn understood ?
  Kratz et al. (since 1993)

➢ importance of $\nu g_{7/2} \rightarrow \pi g_{9/2}$ GT
➢ position of $\nu g_{7/2}$ SP state
$\nu d_{3/2}$ rel. to $\nu h_{11/2}$
➢ spin-orbit splitting $\nu 3p_{3/2}$ - $\nu 3p_{1/2}$
$\nu 2f_{7/2}$ - $\nu 2f_{5/2}$
$\pi 2p_{3/2}$ - $\pi 2p_{1/2}$
$\pi 1f_{7/2}$ - $\pi 1f_{5/2}$
➢ N=82 shell quenching

QRPA (Nilsson, Woods-Saxon,Folded Yukawa)
OXBASH

Nuclear models to calculate $T_{1/2}$ and P_n – (I)

Theoretically,

the two gross/ integral β-decay quantities, $T_{1/2}$ and P_n, are interrelated via their usual definiton in terms of the so-called

β-strength function [$S_\beta(E)$]

What is that?

… a natural adoption of the strength function concept employed in other areas of nuclear physics,

e.g.:	single-particle strength functions,
	s-, p-wave neutron strength functions,
	multipole strength functions for photons.

$$\boxed{S_{\lambda c} = \langle\gamma^2_{\lambda c}\rangle\, \rho_\lambda}$$

$S_{\lambda c}$	refers to the behavior of the squares of overlap integrals ($\gamma^2_{\lambda c}$) between two sets of nuclear wave functions:
l	represents various states of excitation, classified by E, J^π, T;
c	refers to the different reaction / decay channels, classified by E_{part}, l_{part},…
ρ_λ	is the density of levels λ.

Nuclear models to calculate $T_{1/2}$ and P_n – (II)

Application to β-decay:

"Theoretical" definition (Yamada & Takahashi, 1972)

$$\boxed{S_\beta(E) = D^{-1} \cdot |M(E)|^2 \cdot \omega(E) \; [s^{-1}MeV^{-1}]}$$

$\lvert M(E)\rvert$	average β-transition matrix element
$\omega(E)$	level density
D	const., determines Fermi coupling constant g_v^2

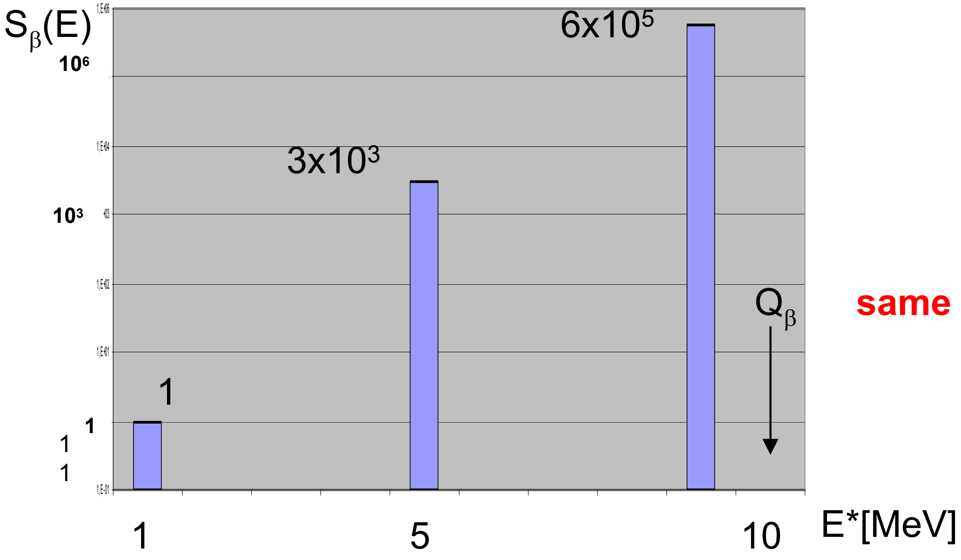
"Experimental" definition (Duke et al., 1970)

$$\boxed{S_\beta(E) = \frac{b(E)}{f(Z, Q_\beta - E) \cdot T_{1/2}} \quad [s^{-1}MeV^{-1}]}$$

b(E)	absolute β-feeding per MeV,
$f(Z, Q_\beta - E)$	Fermi function,
$T_{1/2}$	β–decay half-life.

$T_{1/2}$ as reciprocal ft-value per MeV

$$\boxed{T_{1/2} = \frac{1}{\sum_{0 \le E_i \le Q_\beta} S_\beta(E_i) \times f(Z, Q_\beta - E_i)}}$$



$f(Z, Q_\beta - E_i) \sim (Q_\beta - E_i)^5$ Fermi function

$T_{1/2}$ sensitive to lowest-lying resonances in $S_\beta(E_i)$
P_n sensitive to resonances in $S_\beta(E_i)$ just beyond S_n

same $T_{1/2}$!

↷ easily "correct" $T_{1/2}$ with wrong $S_\beta(E)$

Nuclear models to calculate $T_{1/2}$ and P_n – (III)

Before any theoretical approach is applied, its significance and sophistication should be clear !

In general, 2 groups of models:

(1) “Models” where the physical quantity of interest is given by a polynomial or some other algebraic expression.

- parameters adjusted to exp. data
- describes only a single nucl. property
- no nuclear wave functions
- no insight into underlying SP structure

Examples:

Kratz-Herrmann Formula (1973)
Gross Theory (1973)
.
.
.
New exponential law for $T_{1/2}(\beta^+)$
(Zhang & Ren; 2006)
$T_{1/2}(\beta^-)$ from GTGR + known log(ft)’s
(Kar, Chakravarti & Manfredi; 2006)

(2) Models that use an effective nuclear interaction and solve the microscopic, quantum-mechanical Schrödinger or Dirac equation.

- provides nuclear wave functions
- within the same framework, describes a number of nucl. properties (e.g. g.s.-shape; E_{sp}, J^π, log(ft), $T_{1/2}$ …)

Examples:

FRDM+QRPA (1997; 2006)
Self-consist. Skyrme-HFB + QRPA
(Engel et al.; 1999)
Large-Scale Shell Model
(Martinez-P. & Langanke; 1999, 2003)
Density-Functional + Finite-Fermi System
(Borzov et al.; 2003)
PN-Relativistic QRPA
(Niksic et al.; 2005)

Nuclear models to calculate $T_{1/2}$ and P_n – (IV)

(1) Simple “statistical” approaches

assumptions:

- β-decay energy is large ($Q_\beta \gtrsim 5$ MeV)
- high level density
- $S_\beta(E)$ is a smooth function of E (e.g. S_β=const.; $S_\beta \sim \rho$ (E));
  is insensitive to nature of final states;
  does not vary significantly for different types of nuclei (ee, o-mass, oo).

The Kratz-Herrmann Formula,
applied to P_n values

$$P_n = \frac{\sum\limits_{S_n \le E_i \le Q_\beta} S_\beta(E_i) \text{ x } f\,(Z, Q_\beta - E_i)}{\sum\limits_{C \le E_i \le Q_\beta} S_\beta(E_i) \text{ x } f\,(Z, Q_\beta - E_i)}$$

$$\xrightarrow{\text{with } S_\beta\text{=const.}}$$

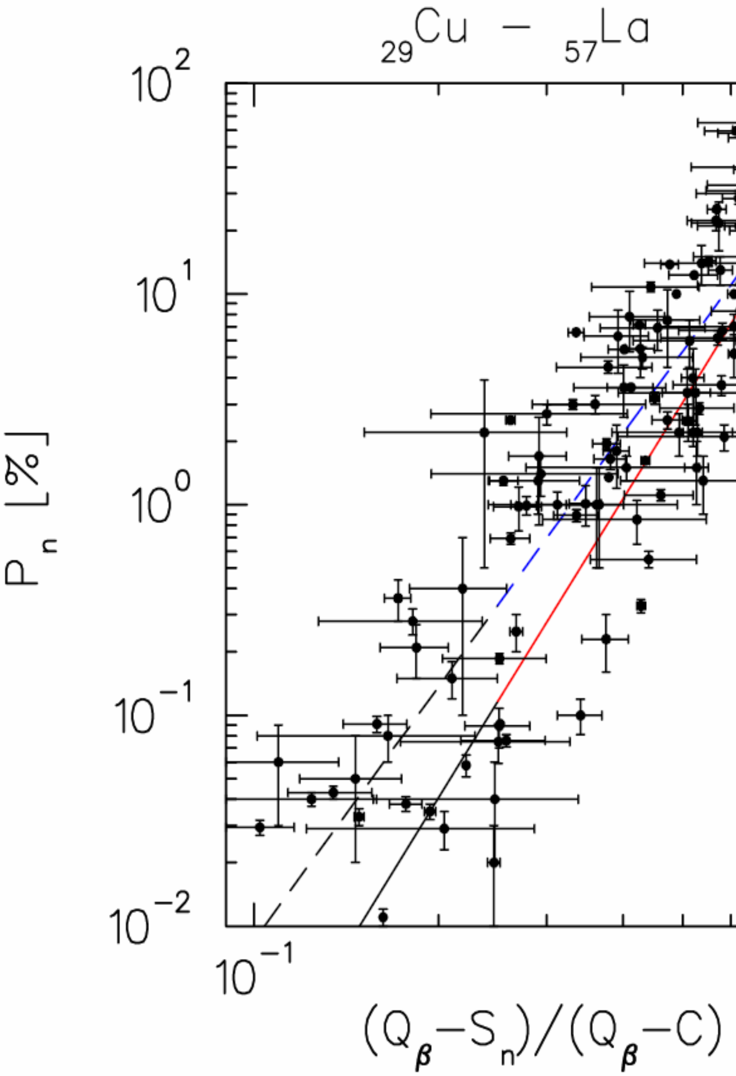
$$P_n \simeq a \left(\frac{(Q_\beta - S_n)}{(Q_\beta - C)} \right)^{b}$$

a, b as “free parameters”, to be determined by a
log-log fit to known P_n-values

C is a “cut-off parameter”
(↷ pairing-gap in β-decay daughter)

Nuclear models to calculate $T_{1/2}$ and P_n – (V)

From Pfeiffer, Kratz & Möller,
Prog. Nuclear Energy 41 (2002) 39-62



Parameters from fits to known P_n-values

dashed line

full line

Region	Lin. regression			Least-squares fit		
	a [%]	b	r²	a [%]	b	red. χ²
29 ≤ Z ≤ 43	88.2	4.1	0.81	106 ±40	5.5 ±0.6	81
47 ≤ Z ≤ 57	84.4	3.9	0.86	123 ±41	4.7 ±0.5	57
29 ≤ Z ≤ 57	**85.2**	**4.0**	**0.83**	**81** **±21**	**4.7** **±0.3**	**78**

… as a kind of “joke”:

$T_{1/2} \approx a\,(Q_\beta - C)^{-b}$

Parameters from fit to known $T_{1/2}$ of n-rich nuclei

Lin. regression			Least-squares fit		
a [ms]	b	r²	a [ms]	b	red. χ²
2.74E06	**4.5**	**0.72**	**7.07E05** **±5.33E05**	**4.0** **±0.4**	**1.1E04**

Nuclear models to calculate $T_{1/2}$ and P_n – (VI)

… NO joke !

in 2006, two examples for big steps BACKWARDS :

(I) X. Zhang & Z. Ren; PRC73, 014305
"New exponential law for β^+ decay half-lives of nuclei far from β-stable line"

"…we have discovered a new exponential law for $T_{1/2}(\beta^+)$…as a function of neutron number…"

$$\log_{10} T_{1/2} = a \times N + b$$

authors give fit parameters for a and b,
for (I) different Z-regions
(II) allowed β^+-decay
(III) first-forbidden β^+-decay
(IV) second-forbidden β^+-decay

↷ finally "a simple and accurate formula" emerges:

$$\log_{10} T_{1/2} = (c_1 Z + c_2)\, N + c_3 Z + c_4$$

"…useful to experimental physicists for analyzing β^+-decay data."

(II) K. Kar, S. Chakravarti & V.R. Manfredi;
arXiv: astro-ph/0603517 v1
"Beta-decay rates (115 < A < 140) for r-process nucleosynthesis"

… the xth re-invention of the Gross Theory !

"… shell model results… indicate that the GT strength distribution.. can be taken as a Gaussian."

"…GT strength distributes among 3 different types of final states:
(a) discrete low-lying states with known log ft's;
(b) discrete states above with unknown strengths;
(c) a part of the GT giant resonance (GTGR)."

admitted "problems":
centroid of GTGR
↷ from Bertsch & Esbensen (1987)
width of GTGR
↷ free parameter !

Nuclear models to calculate $T_{1/2}$ and P_n – (VII)

(2) QRPA – type, “microscopic” models

Recent review by J. Engel; Proc. Workshop on The r-Process… ; Seattle (2004); World Scientific

Among “recent theoretical schemes”…

“Some methods emphasize global applicability, others self-consistency, and still others the comprehensive inclusion of nuclear correlations. None of the methods includes all important correlations, however.”

(2.1) FRDM + QRPA

Macroscopic-microscopic mass model FRDM;
Schrödinger equation solved in QRPA:
GT force

$$\mathbf{V_{GT}} = \chi_{GT} : \vec{\sigma\tau}_{-} \cdot \vec{\sigma\tau}_{+}$$

with “standard choice” for GT interaction

$$\chi_{GT} = \mathbf{23\ MeV/A}$$

latest version includes ff-strength from Gross Theory.

❖ disadvantage: not “self consistent”
❖ advantages: global model for all shapes and types of nuclei; large model space

(2.2) Self-consistent Skyrme-HFB + QRPA

Skyrme interaction SKO
↷ reasonable reproduction of energies and strengths of GT resonances;
strength of T=0 $\nu\pi$ pairing “adjusted” to fit known $T_{1/2}$

❖ disadvantages: only spherical shape; only GT; only ν-magic (N=50, 82, 126); Skyrme interaction not good enough to make…decisive improvement
❖ advantage: self-consistency

↷ **$T_{1/2}$ shorter than those from FRDM + QRPA**

Nuclear models to calculate $T_{1/2}$ and P_n – (VIII)

(2.3) Large-scale Shell Model

shell-model code ANTOINE;
restricted, but sufficiently large SP model space,
with residual interaction split into:
(I) monopole part
(II) renormalized G-matrix component
monopole interaction tuned to reproduce exp. spectra;
admitted, that truncated space may still miss some correlations.

❖ disadvantages:
only ν-magic nuclei (N=50, 82, 126);
only GT-decay;
only spherical.

❖ advantages:
several essential correlations included;
treatment of ee and odd-π isotopes.

↷ $T_{1/2}$ even shorter than those of SC-HFB + QRPA

(2.4) Density Functional HFB + QRPA

density-functional / Greens-function-based model
+ finite-Fermi-systems theory;
not quite selfconsistent,
but with well-developed phenomenology.

❖ disadvantage:
only spherical nuclei

❖ advantages:
all types of nuclei (ee, o-mass, oo);
includes ff-strength microscopically.

↷ $T_{1/2}$ (in particular with ff) short

Nuclear models to calculate $T_{1/2}$ and P_n – (IX)

(2.5) Fully consistent relativistic $\pi\nu$-QRPA

use of new density-dependent interaction in relativistic Hartree-Bogoliubov calculations of g.s. and particle-hole channels;
finite-range Gogny D1S interaction for T=1 pairing channel;
inclusion of $\pi\nu$ particle-particle interaction.

- disadvantages:
  - only spherical ee nuclei;
  - Ni half-lives overestimated by factor ~ 10 (spherical QRPA "normalized" to deformed $^{66}Fe_{40}$ …!);
  - "… our model predicts that ^{132}Sn is stable against β-decay…" (exp.: $T_{1/2}$=40 s ; Q_β=3.12 MeV).
- advantages:
  - "…theoretical $T_{1/2}$ reproduce the exp. data for Fe, Zn, Cd, and Te…";
  - sufficiently large model space.

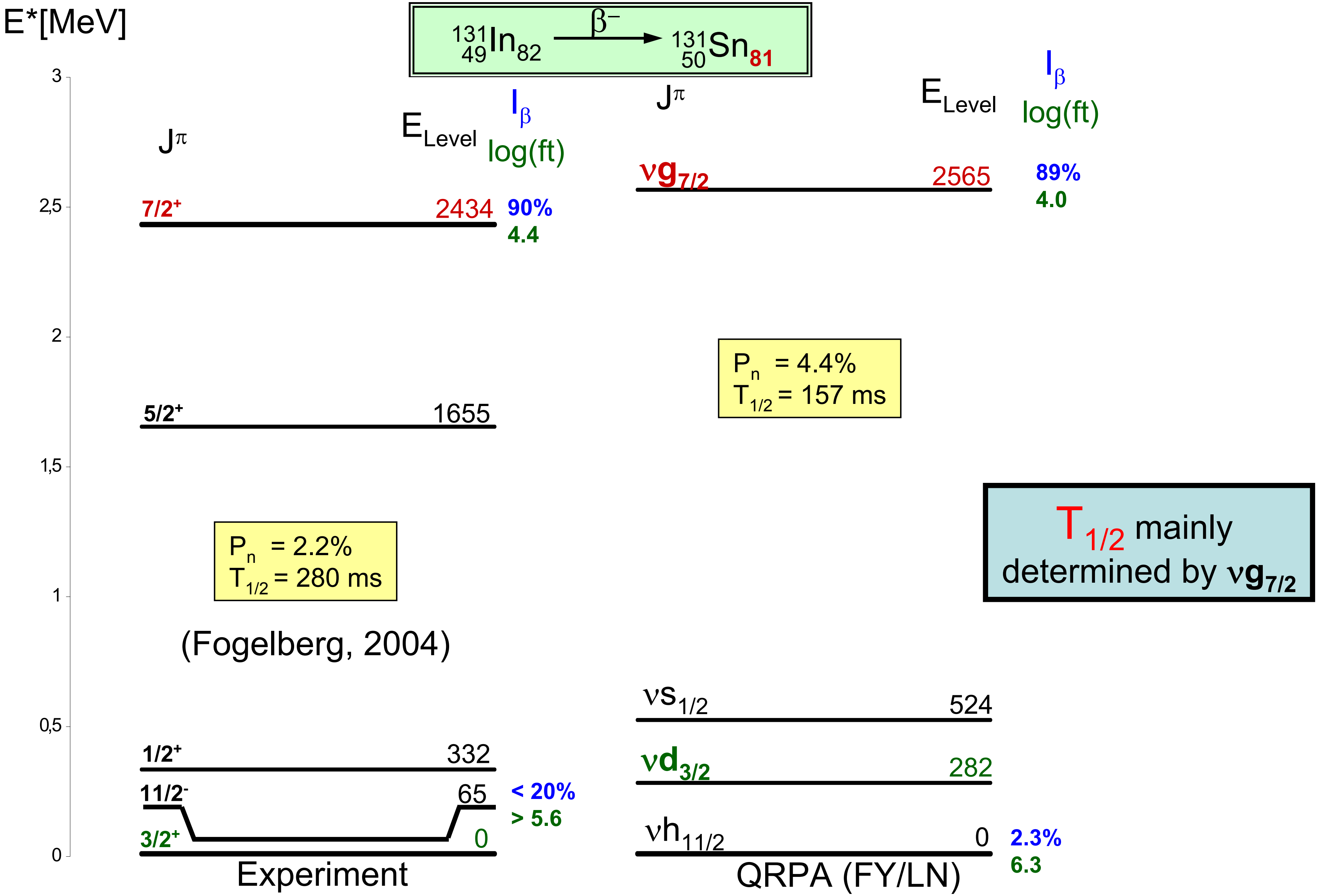
Conclusions

J. Engel
"… it is argued on the basis of a measurement of a strength distribution (i.e. N=82 ^{130}Cd) that the transitions at N=82 calculated by the shell model, HFB + QRPA and Density-functional + FFS are **too fast**.
…this will force the other groups to go back and examine their calculated strength distributions."

P. Möller
"…there is no "correct" model in nuclear physics. Any modeling of nuclear-structure properties involves approximations … to obtain a formulation that can be solved…, but that "retains the essential features" of the true system."

What is known experimentally (I) ?

$^{131}_{49}\mathrm{In}_{82} \xrightarrow{\beta^-} {}^{131}_{50}\mathrm{Sn}_{81}$



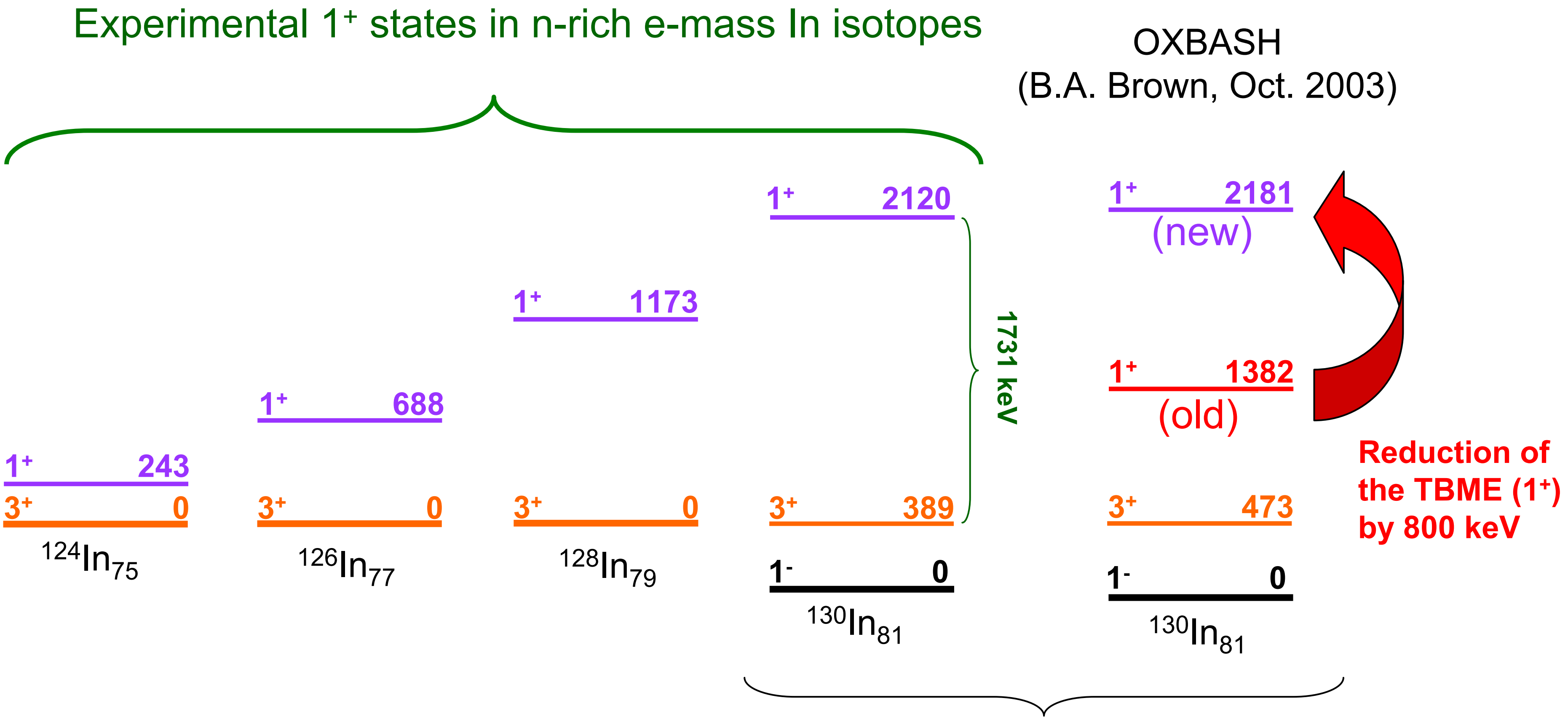
P_n = 2.2%
$T_{1/2}$ = 280 ms

(Fogelberg, 2004)

P_n = 4.4%
$T_{1/2}$ = 157 ms

$T_{1/2}$ mainly determined by $\nu g_{7/2}$

What is known experimentally (II) ?



Dillmann et al., 2003

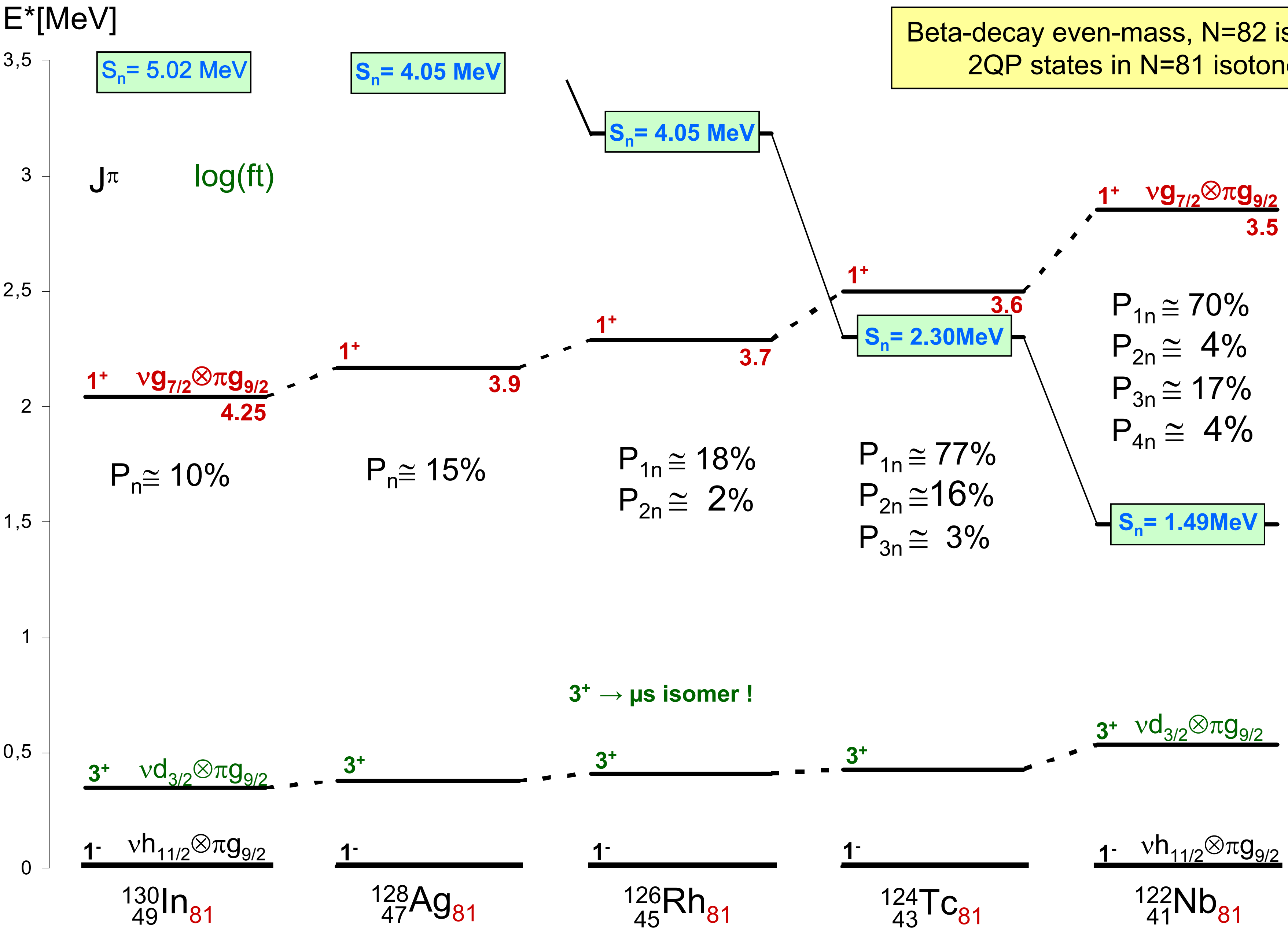
Configuration 1$^-$: $\nu h_{11/2} \otimes \pi g_{9/2}$

Configuration 3$^+$: $\nu d_{3/2} \otimes \pi g_{9/2}$

Configuration 1$^+$: $\nu g_{7/2} \otimes \pi g_{9/2}$ → determines $\mathbf{T_{1/2}}$

Short range extrapolation (I)

Beta-decay even-mass, N=82 isotones
2QP states in N=81 isotones



$P_n \cong 10\%$

$P_n \cong 15\%$

$P_{1n} \cong 18\%$
$P_{2n} \cong 2\%$

$P_{1n} \cong 77\%$
$P_{2n} \cong 16\%$
$P_{3n} \cong 3\%$

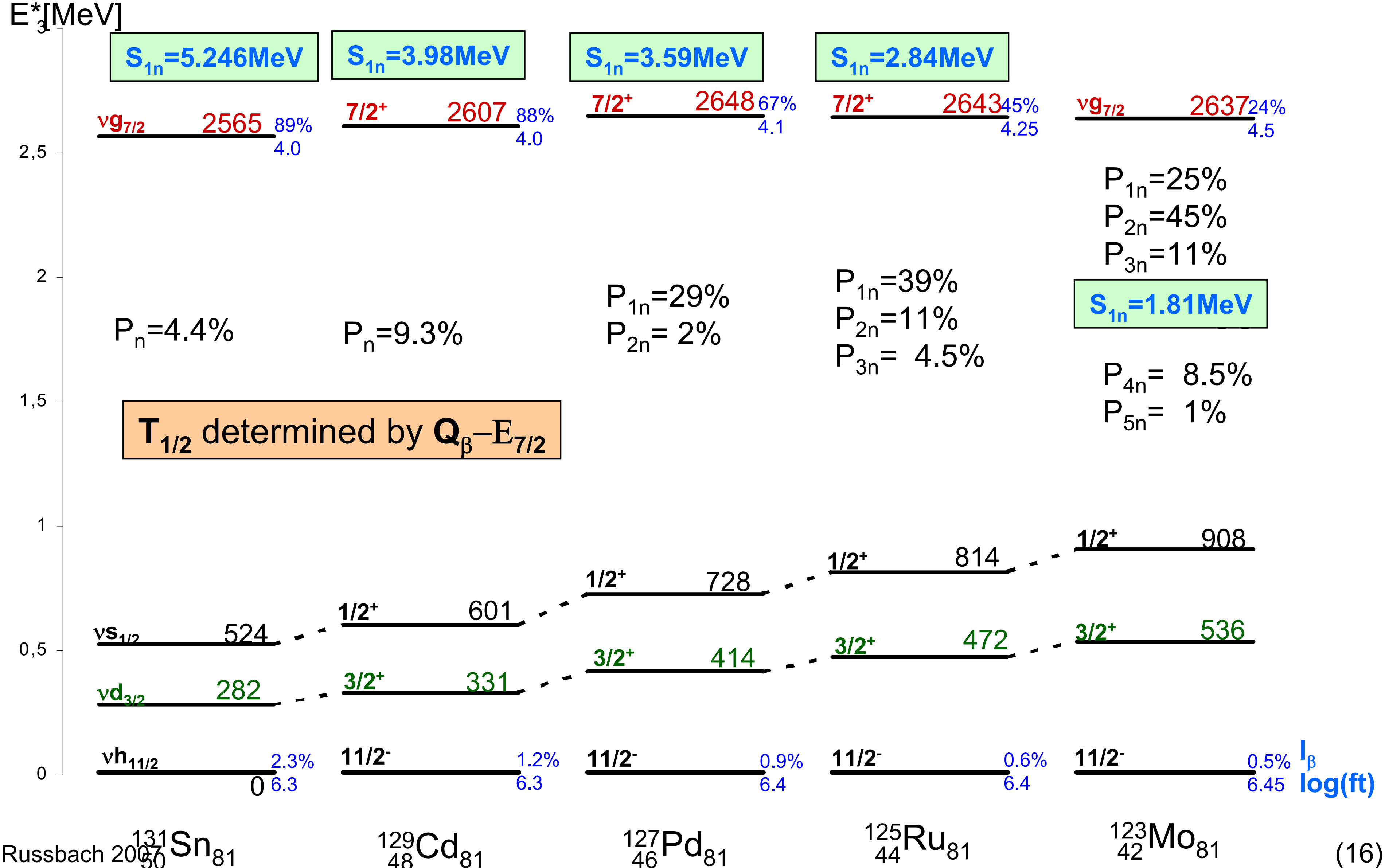
$P_{1n} \cong 70\%$
$P_{2n} \cong 4\%$
$P_{3n} \cong 17\%$
$P_{4n} \cong 4\%$

$3^+ \rightarrow \mu s$ isomer !

Russbach 2007

Short range extrapolation (II)

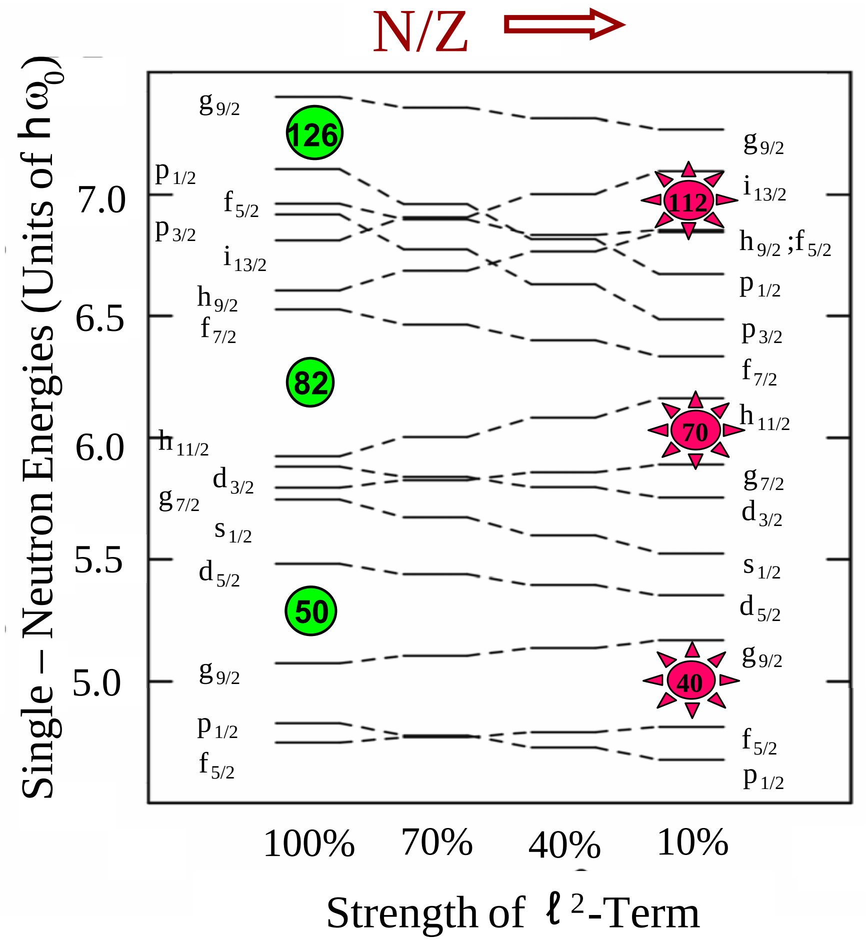
Beta-decay odd-mass, N=82 isotones
νSP states in N=81 isotones



$T_{1/2}$ determined by $Q_\beta - E_{7/2}$

Russbach 2007

Effects of N=82 „shell quenching“



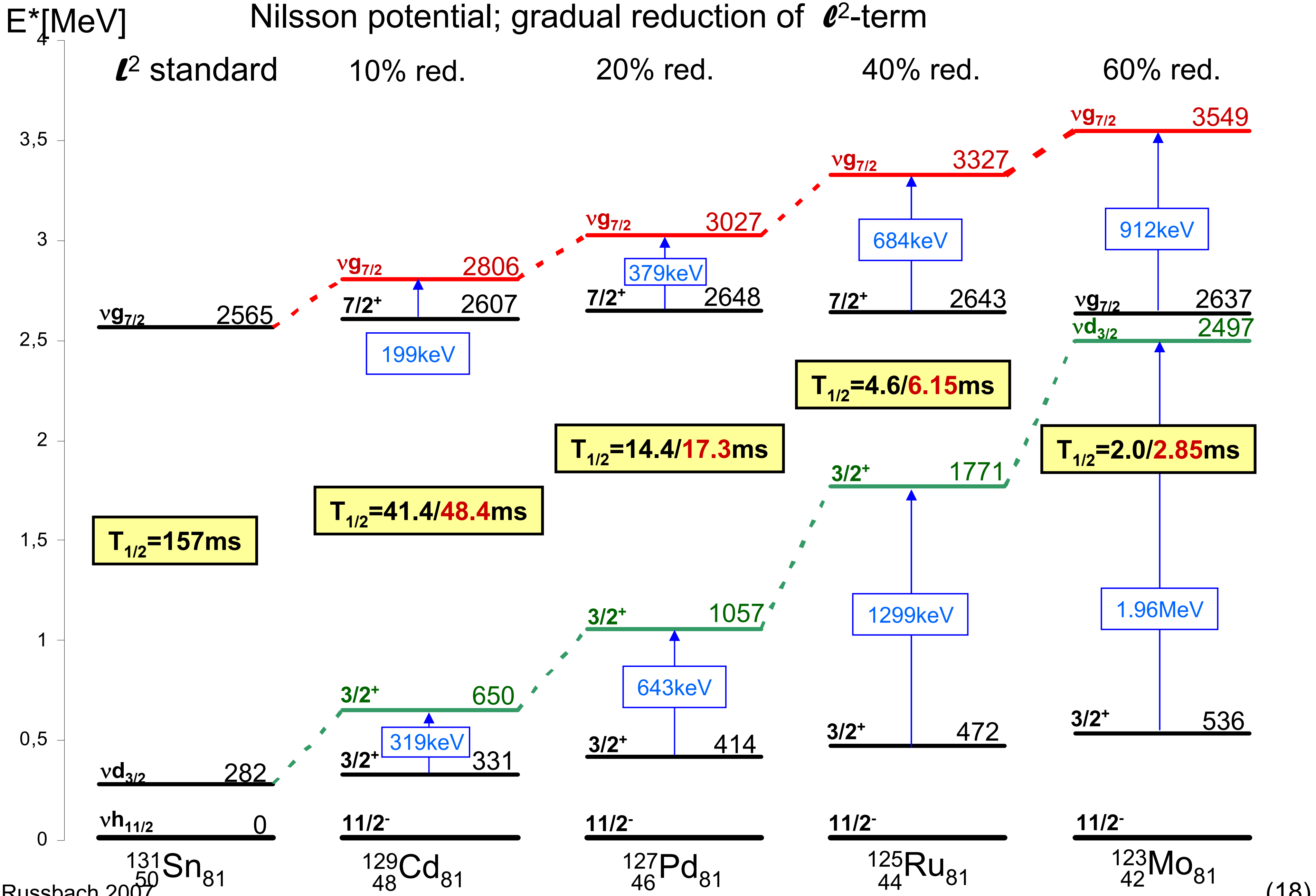
B. Pfeiffer et al.,
Acta Phys. Polon. **B27** (1996)

- high-j orbitals ⇑ (e.g. $\nu h_{11/2}$)
- low-j orbitals ⇓ (e.g. $\nu d_{3/2}$)
- evtl. crossing of orbitals
- new “magic” numbers / shell gaps (e.g. ${}^{110}_{40}\mathrm{Zr}_{\mathbf{70}}$, ${}^{170}_{58}\mathrm{Ce}_{\mathbf{112}}$)

change of $T_{1/2}$?

Possible effects of “shell quenching”

Nilsson potential; gradual reduction of ℓ^2-term



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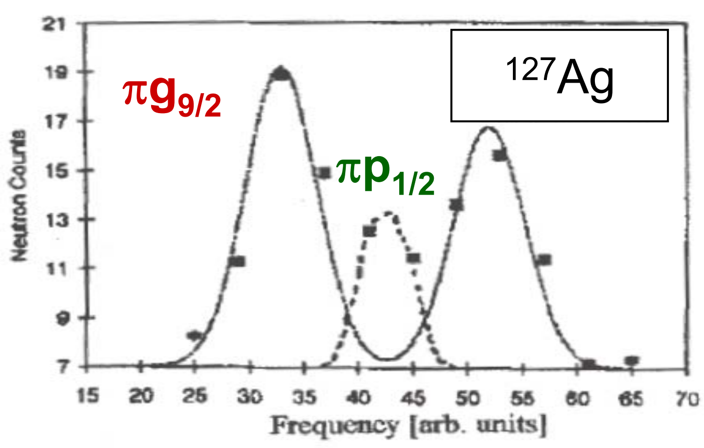
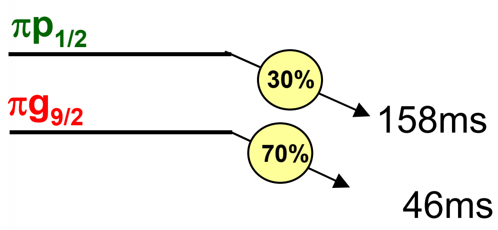
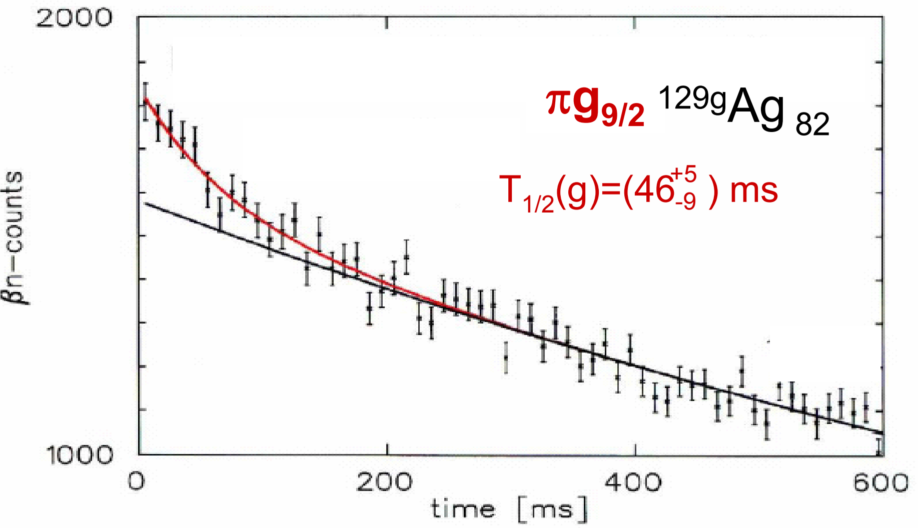
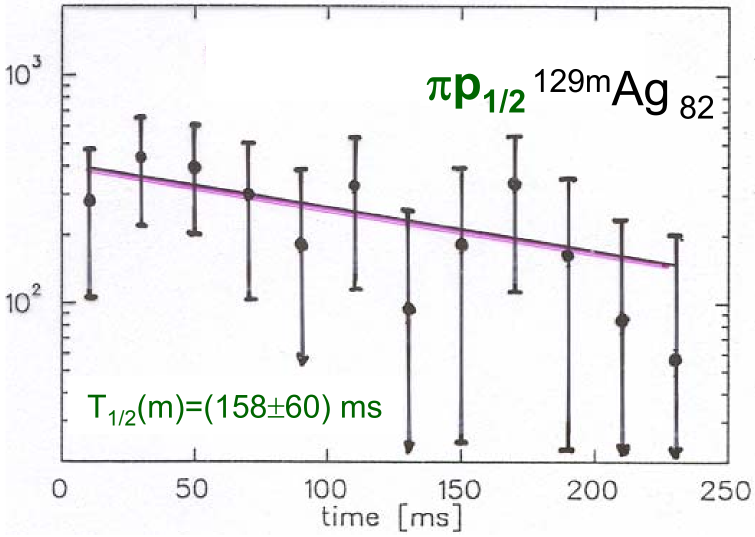
Beta-decay of ^{129}Ag isomers

Separation of isomers
by fine-tuning of
laser frequency









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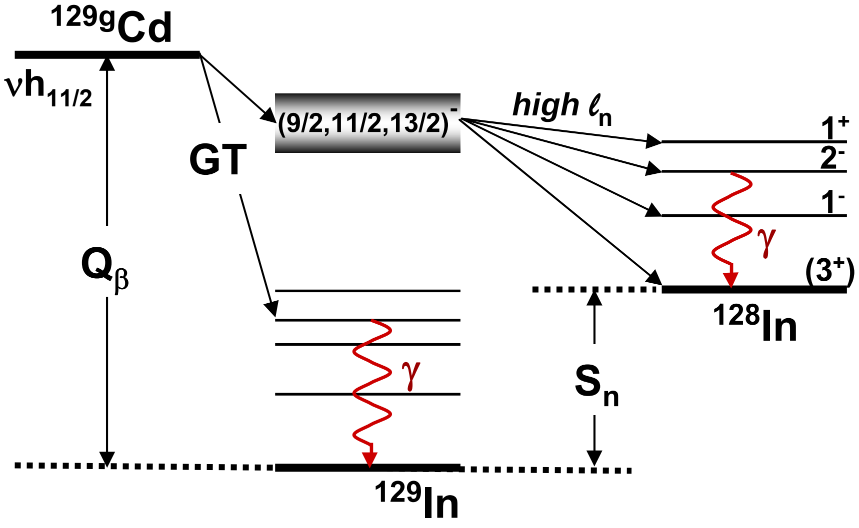
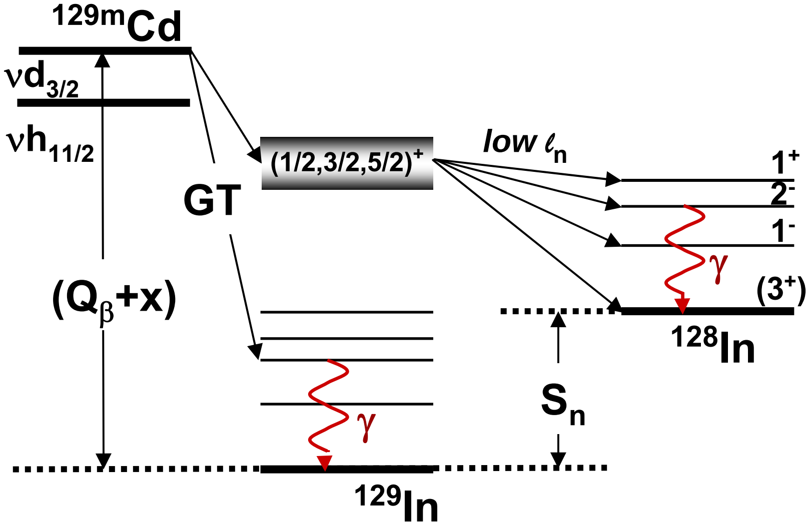
$^{129}Cd_{81}$ βdn - emission



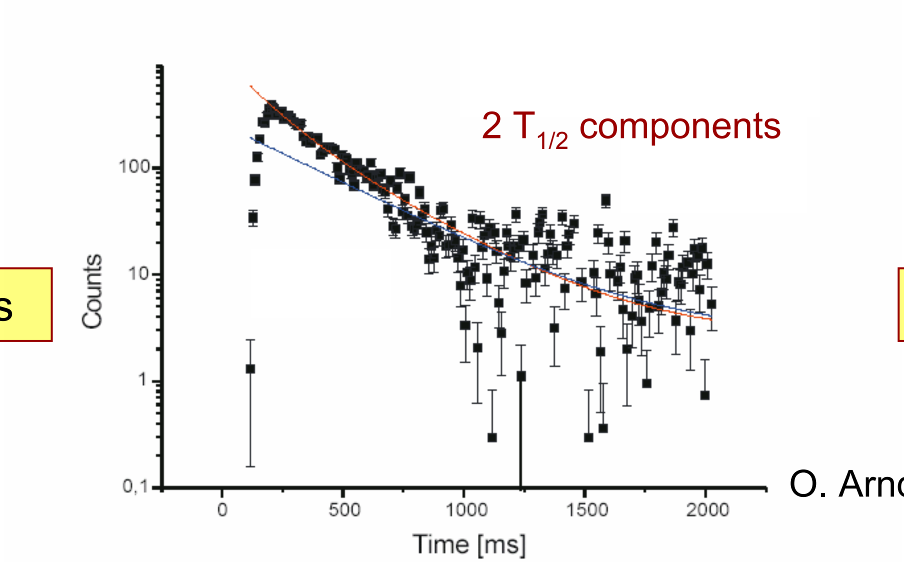
<u>"hard"</u> βdn-spectrum mainly outer ^{3}He ring



<u>"soft"</u> βdn-spectrum mainly inner ^{3}He ring

of neutron longcounter

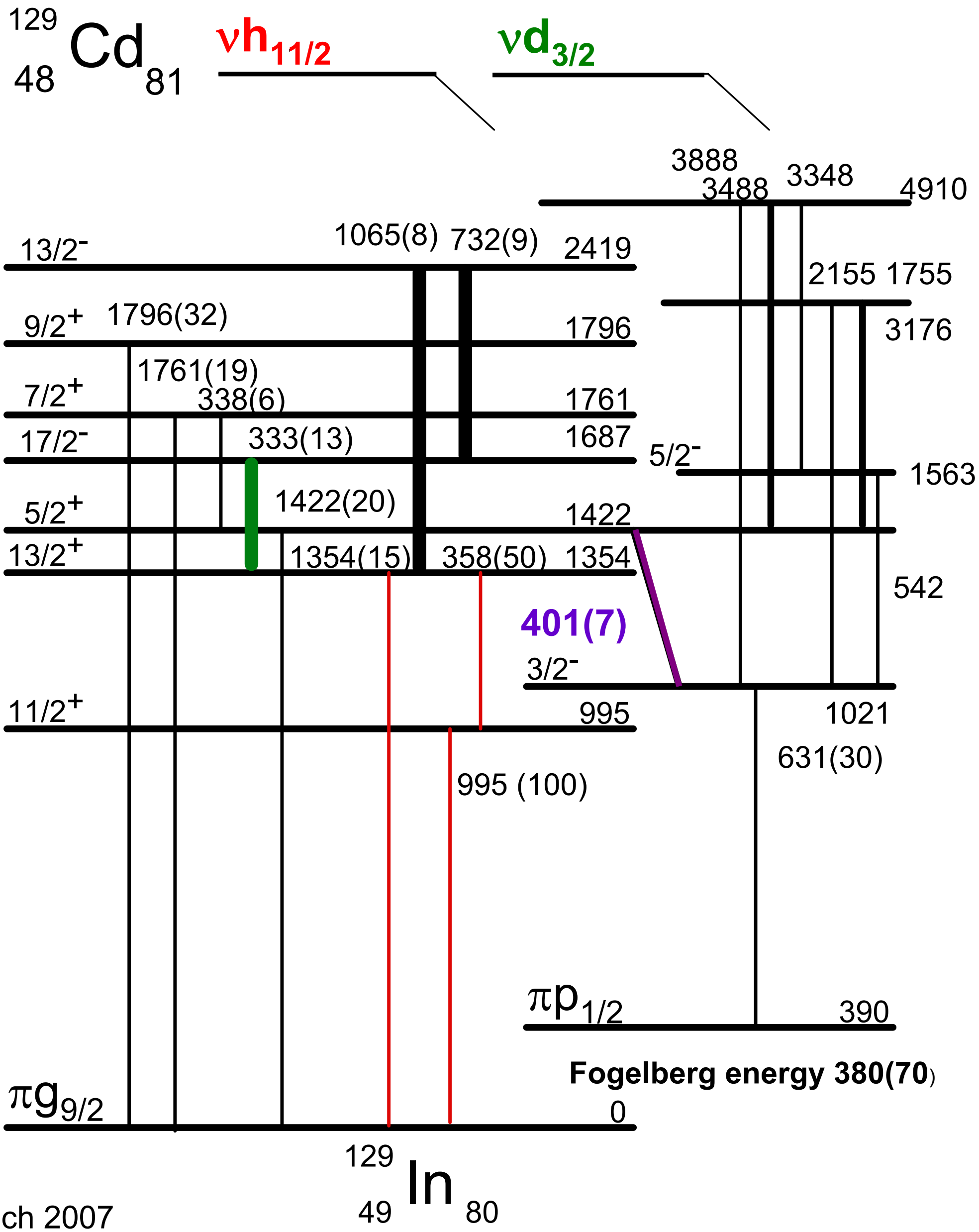
$T_{½}(^{129g}Cd) = 242(8)$ ms



$T_{½}(^{129m}Cd) = 104(6)$ ms

O. Arndt (Diploma Thesis; 2003)

Beta – decay of $^{129}Cd_{81}$ isomers



* search for γ-branching from $\nu h_{11/2}$ g.s.-decay of $^{129}Cd_{81}$ to $\nu d_{3/2}$ isomer decay
$\Rightarrow$ possible candidate (very weak!) 401 keV γ-line

* determine splitting $\pi g_{9/2}$ g.s. and $\pi p_{1/2}$ isomeric state in $^{129}_{49}In$

Terrestrial and stellar half-lives of odd-mass N=82 waiting-point isotopes

Isotope	Experiment			QRPA(GT+ff)*)		
	$T_{1/2}(\pi g_{9/2})$	$T_{1/2}(\pi p_{1/2})$	$T_{1/2}$(stellar)	$T_{1/2}(\pi g_{9/2})$	$T_{1/2}(\pi p_{1/2})$	$T_{1/2}$(stellar)
$^{131}_{49}$In	280ms	350ms	300ms	157ms	477ms	253ms
$^{129}_{47}$Ag	46ms	158ms	80ms	43ms	140ms	72ms
$^{127}_{45}$Rh	------	-----	------	14.4ms	25.4ms	17.7ms
$^{125}_{43}$Tc	------	-----	------	4.60ms	4.45ms	4.5ms
$^{123}_{41}$Nb	------	-----	------	2.01ms	1.91ms	1.98ms

*) Nuclear masses: ADMC,2003 & ETFSI-Q

Let’s come back to global calculations of gross β-decay properties…

… only model that can calculate on a macroscopic-microscopic basis
all types of nuclei
(nearly) all nuclear shapes
g.s. and odd-particle excited-states decays:

PHYSICAL REVIEW C **67**, 055802 (2003)

New calculations of gross β-decay properties for astrophysical applications: Speeding-up the classical r process

Peter Möller
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Bernd Pfeiffer and Karl-Ludwig Kratz
Institut für Kernchemie, Universität Mainz, Germany

mass models:	FRDM	(ADNDT 59, 1995)
	ETFSI-Q	(PLB 387, 1996)
QRPA model:	pure GT	(ADNDT 66, 1997)
	GT + ff	(see above; URL: http://t16web/moeller/publications/rspeed2002.html; ADNDT, to be submitted; KCh Mainz Report (unpubl.), URL: www.kernchemie.uni-mainz.de)

$T_{1/2}$ and P_n calculations in 3 steps – (I)

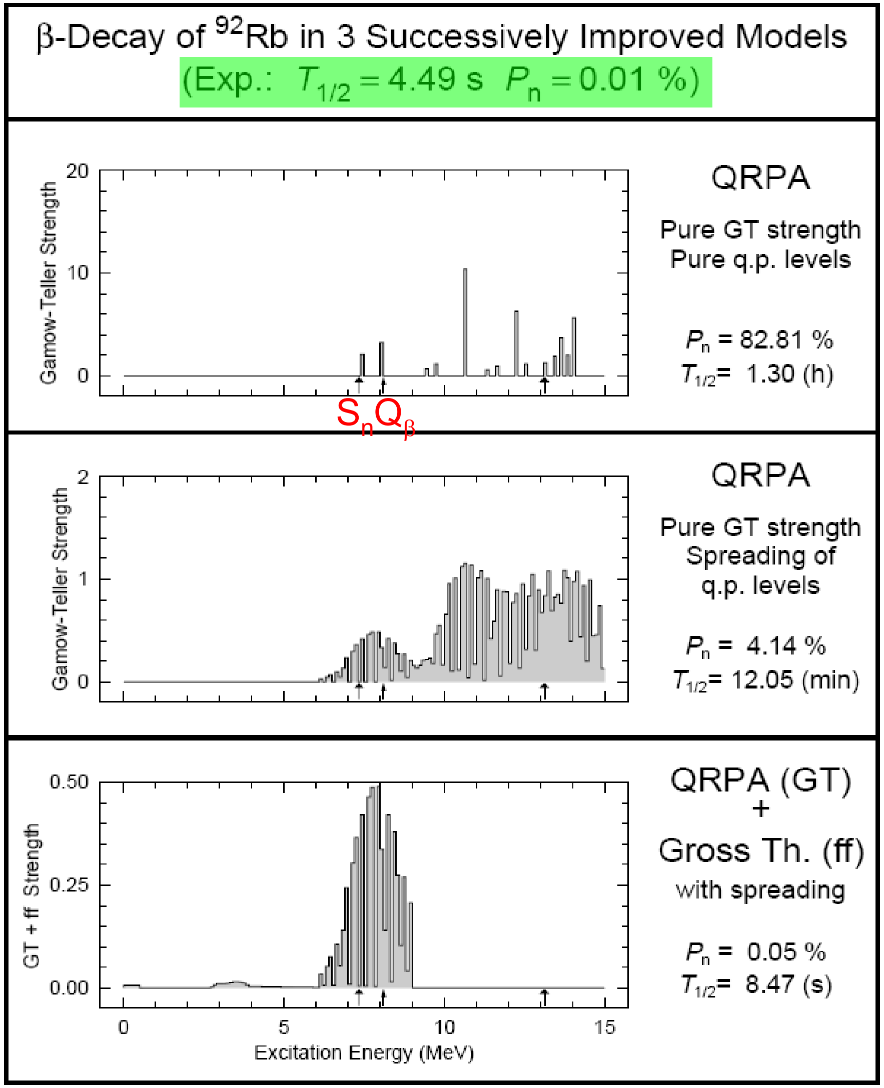
"Typical example":

(1) FRDM /ETFSI-Q
↷ Q_β, S_n, ε_2
Folded-Yukawa wave fcts.

QRPA pure GT
with input from mass model
potential: Folded Yukawa
Nilsson (different κ, μ)
Woods-Saxon
pairing-model: Lipkin-Nogami
BCS

(2) as in (1) with empirical spreading of SP transition strength, as shown in experimental $S_\beta(E)$

(3) as in (2) with addition of first-forbidden strength from Gross Theory



β-Decay of ^{92}Rb in 3 Successively Improved Models
(Exp.: $T_{1/2} = 4.49$ s $P_n = 0.01$ %)

QRPA
Pure GT strength
Pure q.p. levels
P_n = 82.81 %
$T_{1/2}$ = 1.30 (h)

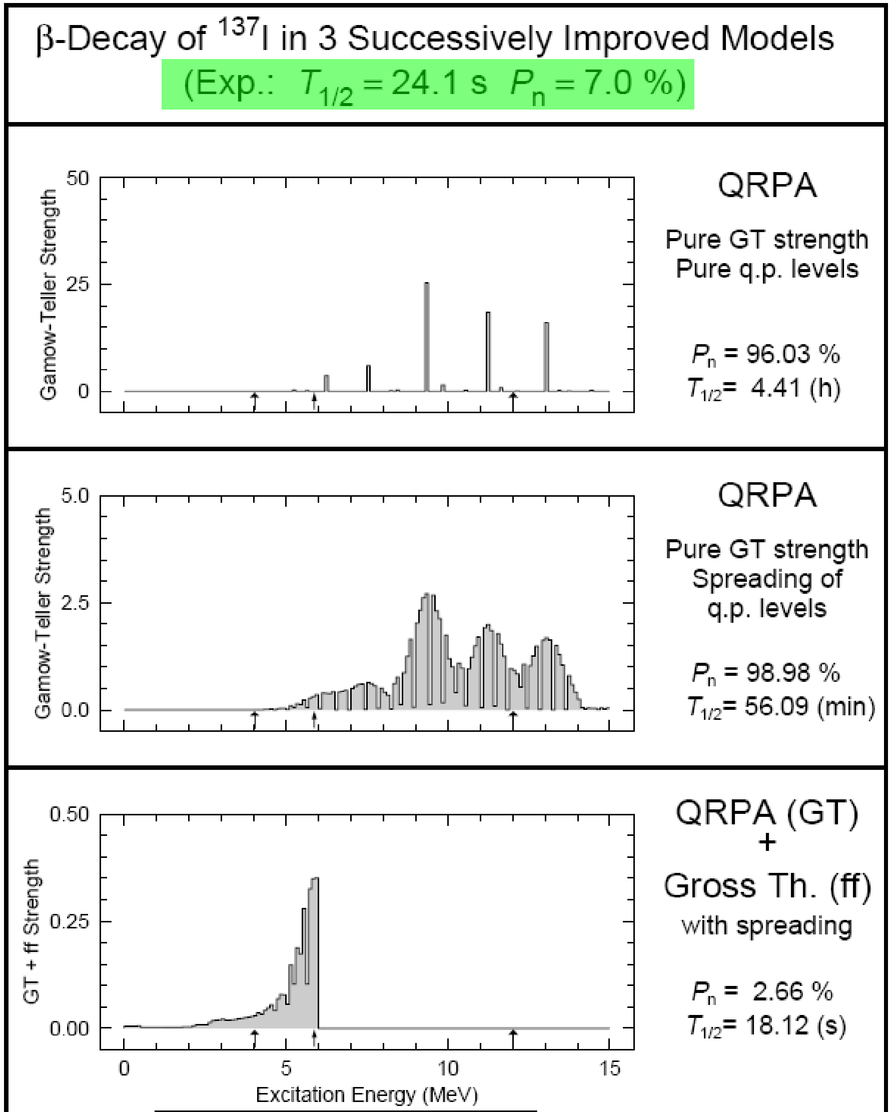
QRPA
Pure GT strength
Spreading of q.p. levels
P_n = 4.14 %
$T_{1/2}$ = 12.05 (min)

QRPA (GT)
+
Gross Th. (ff)
with spreading
P_n = 0.05 %
$T_{1/2}$ = 8.47 (s)

note: effect on P_n !

$T_{1/2}$ and P_n calculations in 3 steps – (II)

Another “spherical” case:

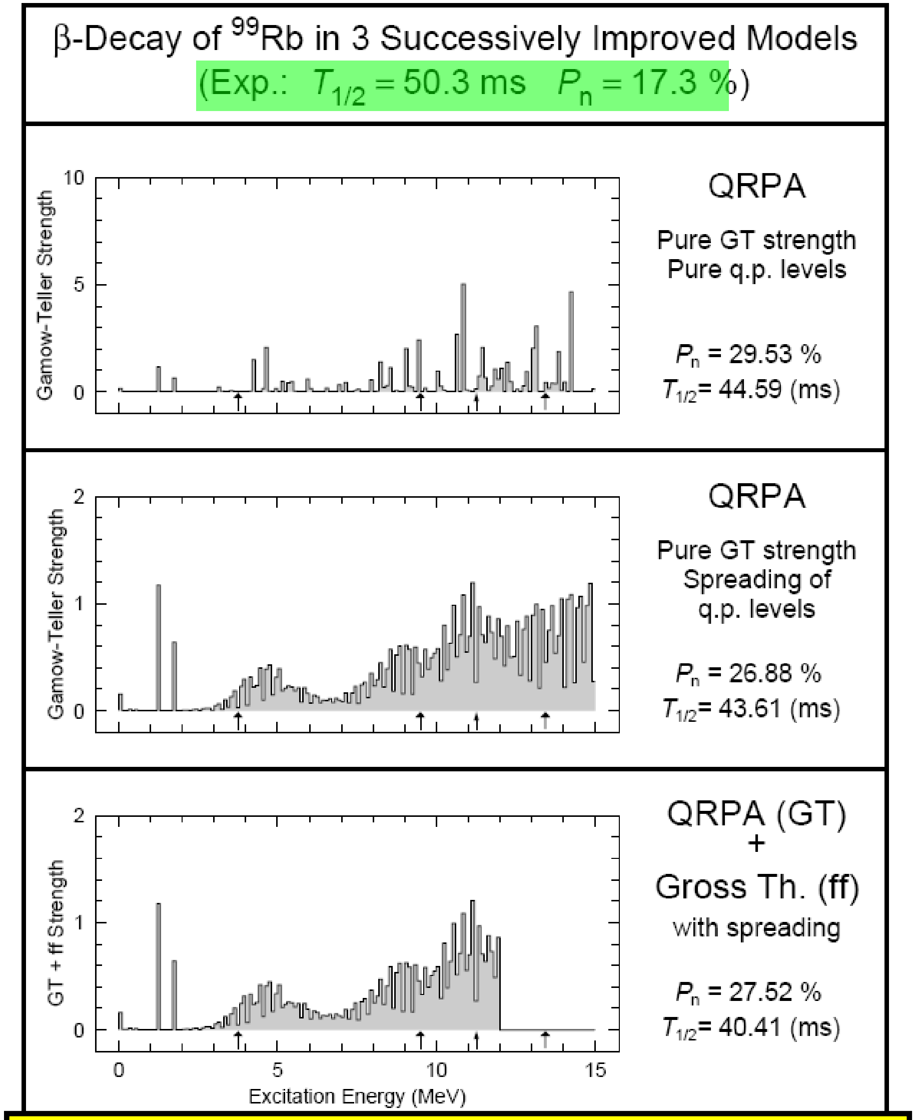


β-Decay of ^{137}I in 3 Successively Improved Models

(Exp.: $T_{1/2} = 24.1$ s $P_n = 7.0$ %)

note : effect on $T_{1/2}$!

…and a typical “deformed” case:



β-Decay of ^{99}Rb in 3 Successively Improved Models

(Exp.: $T_{1/2} = 50.3$ ms $P_n = 17.3$ %)

Note: low-lying GT-strength; ff-strength unimportant!

Experimental vs. theoretical β-decay properties

$T_{1/2}$, P_n ⟶ gross **β-strength** properties from FRDM + QRPA

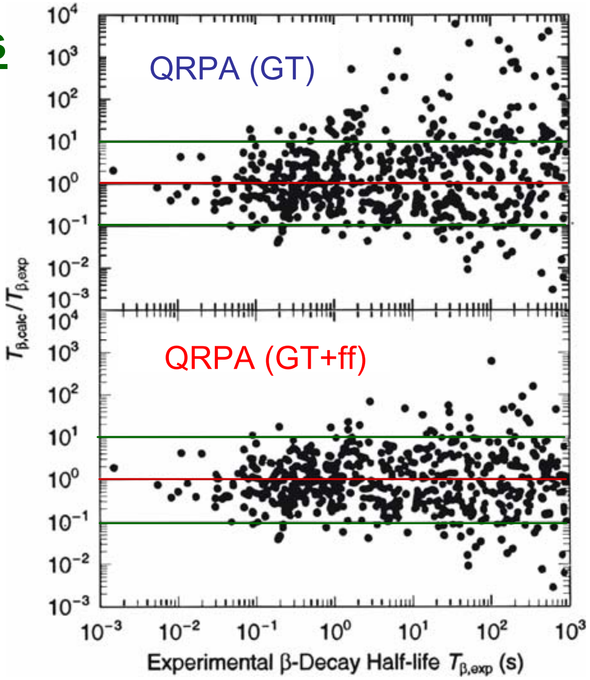
Requests: (I) prediction / reproduction of correct experimental "**number**"

(II) detailed nuclear-structure understanding

↷ full spectroscopy of "key" isotopes, like $^{80}\mathrm{Zn}_{50}$, $^{130}\mathrm{Cd}_{82}$.

Half-lives

Total Error = 3.73

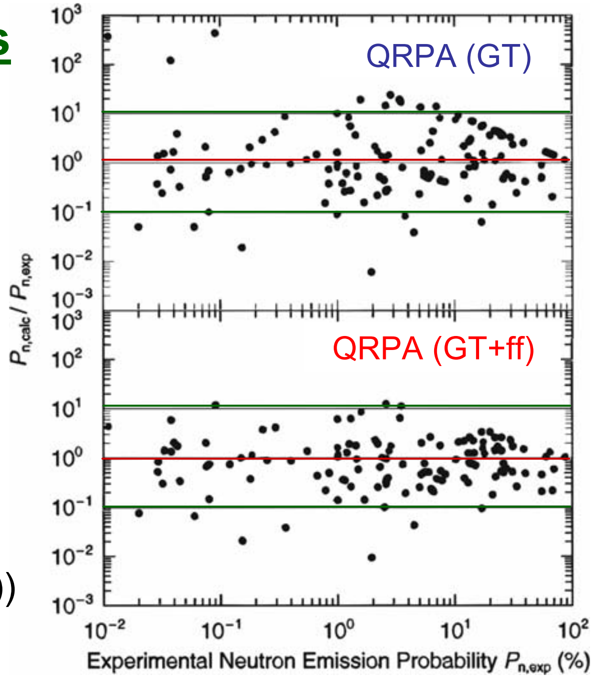


Total Error = 3.08

(P. Möller et al.,
PR **C67**, 055802 (2003))

P_n-Values

Total Error = 5.54

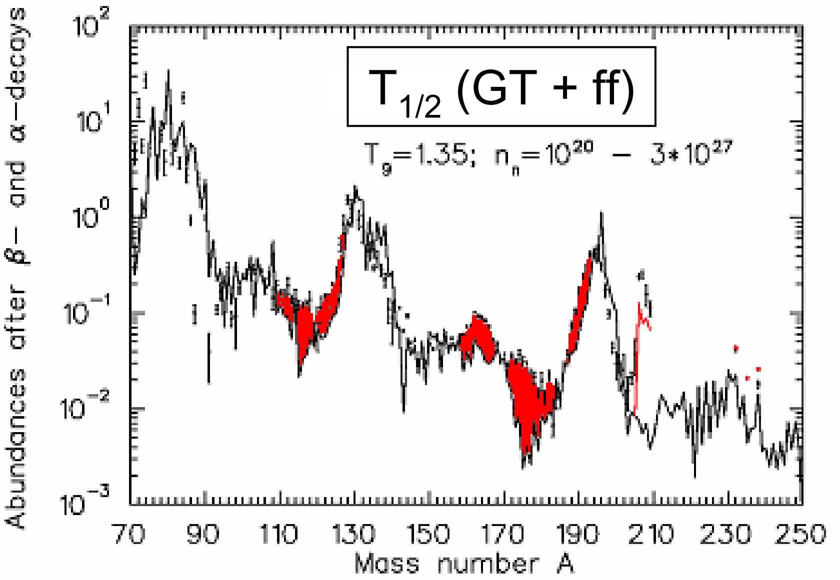


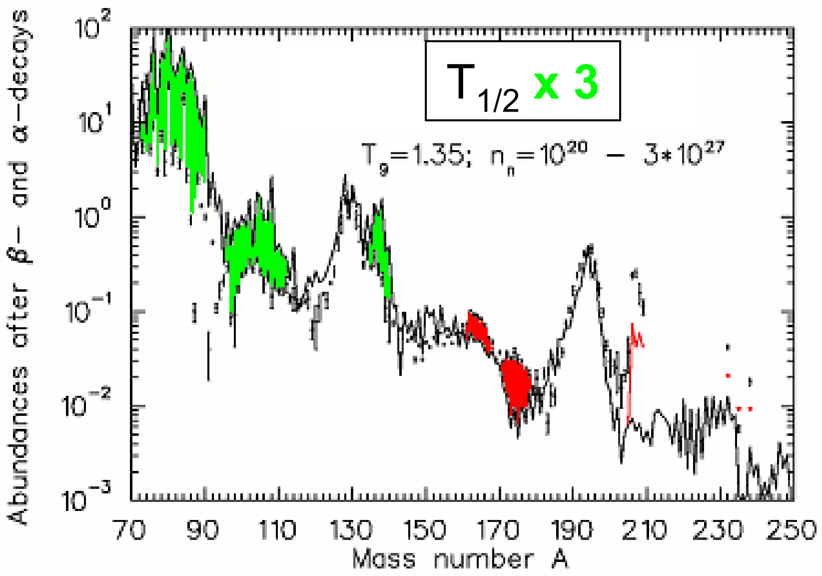
Total Error = 3.52

Effects of $T_{1/2}$ on r-process matter flow

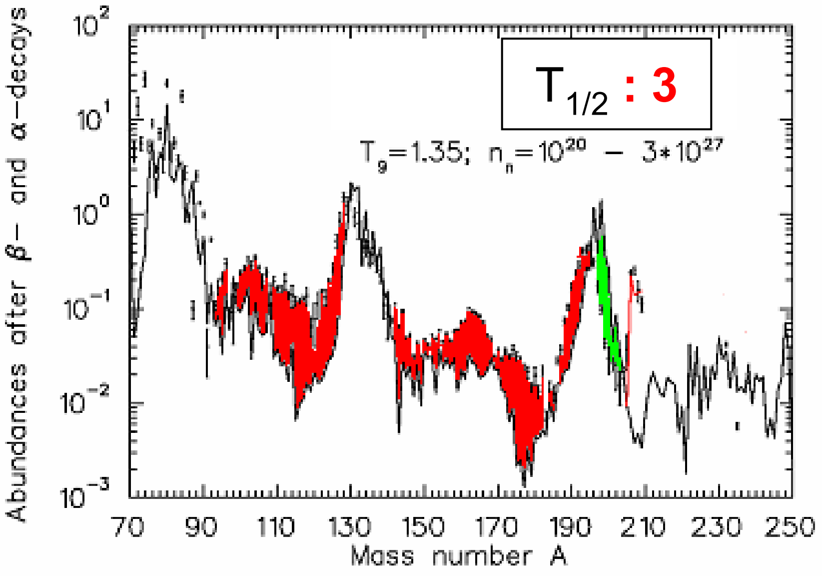
Mass model: **ETFSI-Q**
- all astro-parameters kept constant

r-process model:
“waiting-point approximation“





r-matter flow too **slow**



r-matter flow too **fast**

Most recent Publications

T. Marketin, D. Vretenar and P. Ring;
Calculation of β-decay rates in a relativistic model with momentum-dependent self energies;
Phys. Rev. C 75 (2007) 024304

N. Costiris, E. Mavrommatis, K.A. Gernoth and J.W. Clark;
A Global Model of β-Decay Half-Lives Using Neural Networks;
arXiv:nucl-th/0701096v1 (Jan. 2007)

I.N. Borzov;
Beta-decay rates;
Nucl. Phys. A 777 (2006) 645

Astrophysical consequences

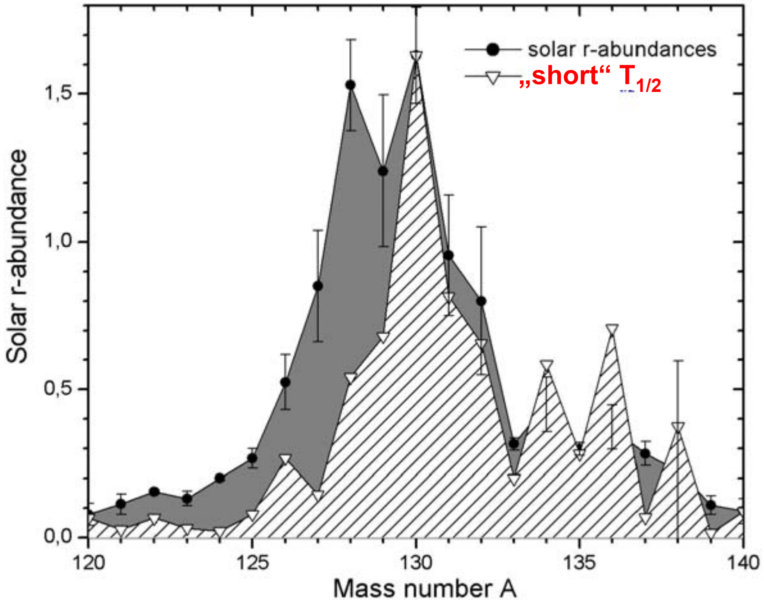
Longer $T_{1/2}$!

... resulting from new experimental and theoretical nuclear structure information:

- better understanding of formation and shape of, as well as r-process matter flow through the A≈130 $N_{r,\odot}$ peak
- no justification to question waiting-point concept (Langanke et al., PRL 83, 199; Nucl. Phys. News 10, 2000)
- no need to request sizeable effects from ν-induced reactions (Qian et al., PRC 55, 1997)

⇨ r-process abundances in the Solar System and in UMP Halo stars...

...are governed by nuclear structure!



Nuclear masses from

AMDC, 2003

ETFSI-Q

Normalized to $N_{r,\odot}$ (^{130}Te)